Solutions Manual

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CHAPTER 1 PROBLEMS

1.1 The demand estimation is the starting point for planning the future electric power supply. The consistency of demand growth over the years has led to numerous attempts to fit mathematical curves to this trend. One of the simplest curves is

$$P = P_0 e^{a(t-t_0)}$$

where a is the average per unit growth rate, P is the demand in year t, and P_0 is the given demand at year t_0.

Assume the peak power demand in the United States in 1984 is 480 GW with an average growth rate of 3.4 percent. Using *MATLAB*, plot the predicated peak demand in GW from 1984 to 1999. Estimate the peak power demand for the year 1999.

We use the following commands to plot the demand growth

```
t0 = 84; P0 = 480;
a =.034;
t =(84:1:99)';
P =P0*exp(a*(t-t0));
disp('Predicted Peak Demand - GW')
disp([t, P])
plot(t, P), grid
xlabel('Year'), ylabel('Peak power demand GW')
P99 =P0*exp(a*(99 - t0))
```

The result is

```
Predicted Peak Demand - GW
   84.0000  480.0000
   85.0000  496.6006
   86.0000  513.7753
   87.0000  531.5441
   88.0000  549.9273
   89.0000  568.9463
   90.0000  588.6231
   91.0000  608.9804
   92.0000  630.0418
   93.0000  651.8315
   94.0000  674.3740
   95.0000  697.6978
   96.0000  721.8274
   97.0000  746.7916
   98.0000  772.6190
   99.0000  799.3398

P99 =

      799.3398
```

The plot of the predicated demand is shown n Figure 1.

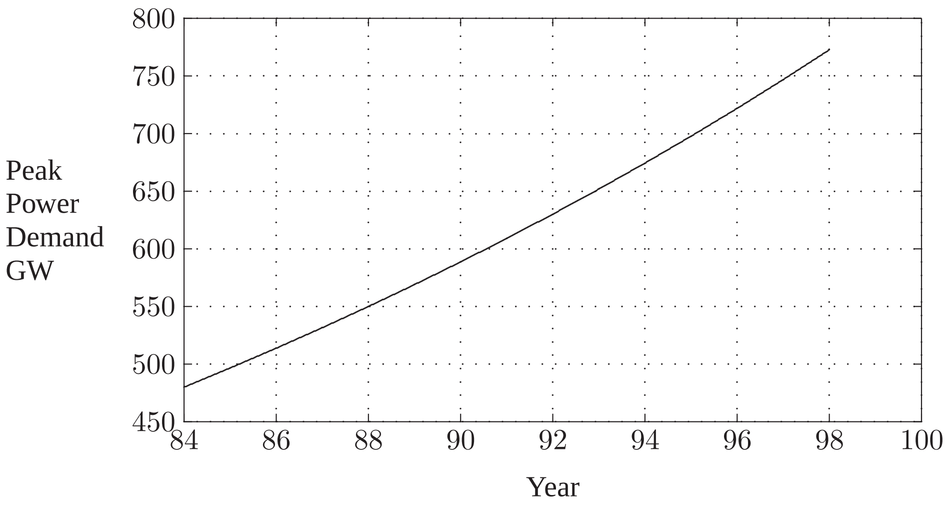


FIGURE 1
Peak Power Demand for Problem 1.1

1.2 In a certain country, the energy consumption is expected to double in 10 years.

Assuming a simple exponential growth given by

$$P = P_0 e^{at}$$

calculate the growth rate a.

$$\begin{aligned} 2P_0 &= P_0 e^{10a} \\ \ln 2 &= 10a \end{aligned}$$

Solving for a, we have

$$a = \frac{0.693}{10} = 0.0693 = 6.93\%$$

1.3. The annual load of a substation is given in the following table. During each month, the power is assumed constant at an average value. Using *MATLAB* and the **barcycle** function, obtain a plot of the annual load curve. Write the necessary statements to find the average load and the annual load factor.

Annual System Load	
Interval – Month	Load – MW
January	8
February	6
March	4
April	2
May	6
June	12
July	16
August	14
September	10
October	4
November	6
December	8

The following commands

```
data = [ 0   1    8
         1   2    6
         2   3    4
         3   4    2
         4   5    6
         5   6   12
```

```
            6   7   16
            7   8   14
            8   9   10
            9  10    4
           10  11    6
           11  12    8];
P = data(:,3);                               % Column array of load
Dt = data(:, 2) - data(:,1); % Column array of demand interval
W = P'*Dt;                     % Total energy, area under the curve
Pavg = W/sum(Dt)                                     % Average load
Peak = max(P)                                           % Peak load
LF = Pavg/Peak*100                           % Percent load factor
barcycle(data)                               % Plots the load cycle
xlabel('time, month'), ylabel('P, MW'), grid
```

result in

```
    Pavg =
             8
    Peak =
            16
    LF =
            50
```

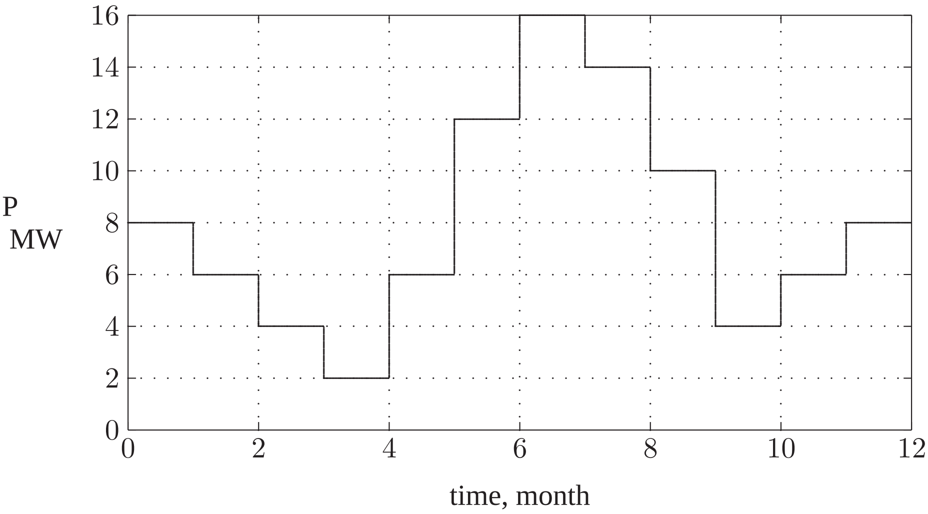


FIGURE 2
Monthly load cycle for Problem 1.3

CHAPTER 2 PROBLEMS

2.1. Modify the program in Example 2.1 such that the following quantities can be entered by the user:
The peak amplitude V_m, and the phase angle θ_v of the sinusoidal supply $v(t) = V_m \cos(\omega t + \theta_v)$. The impedance magnitude Z, and its phase angle γ of the load.
The program should produce plots for $i(t)$, $v(t)$, $p(t)$, $p_r(t)$ and $p_x(t)$, similar to Example 2.1. Run the program for $V_m = 100$ V, $\theta_v = 0$ and the following loads:

An inductive load, $Z = 1.25\angle 60^\circ\ \Omega$
A capacitive load, $Z = 2.0\angle -30^\circ\ \Omega$
A resistive load, $Z = 2.5\angle 0^\circ\ \Omega$

(a) From $p_r(t)$ and $p_x(t)$ plots, estimate the real and reactive power for each load. Draw a conclusion regarding the sign of reactive power for inductive and capacitive loads.
(b) Using phasor values of current and voltage, calculate the real and reactive power for each load and compare with the results obtained from the curves.
(c) If the above loads are all connected across the same power supply, determine the total real and reactive power taken from the supply.

The following statements are used to plot the instantaneous voltage, current, and the instantaneous terms given by(2-6) and (2-8).

```
Vm = input('Enter voltage peak amplitude Vm = ');
thetav =input('Enter voltage phase angle in degree thetav = ');
Vm = 100; thetav = 0;      % Voltage amplitude and phase angle
Z = input('Enter magnitude of the load impedance  Z = ');
gama = input('Enter load phase angle in degree gama = ');
thetai = thetav - gama;        % Current phase angle in degree
```

```
theta = (thetav - thetai)*pi/180;          % Degree to radian
Im = Vm/Z;                                 % Current amplitude
wt=0:.05:2*pi;                             % wt from 0 to 2*pi
v=Vm*cos(wt);                          % Instantaneous voltage
i=Im*cos(wt + thetai*pi/180);          % Instantaneous current
p=v.*i;                                  % Instantaneous power
V=Vm/sqrt(2); I=Im/sqrt(2);          % RMS voltage and current
pr = V*I*cos(theta)*(1 + cos(2*wt));                % Eq. (2.6)
px = V*I*sin(theta)*sin(2*wt);                      % Eq. (2.8)
disp('(a) Estimate from the plots')
P = max(pr)/2, Q = V*I*sin(theta)*sin(2*pi/4)
P = P*ones(1, length(wt));            % Average power for plot
xline = zeros(1, length(wt));        % generates a zero vector
wt=180/pi*wt;                    % converting radian to degree
subplot(221), plot(wt, v, wt, i,wt, xline), grid
title(['v(t)=Vm coswt, i(t)=Im cos(wt +',num2str(thetai),')'])
xlabel('wt, degrees')
subplot(222), plot(wt, p, wt, xline), grid
title('p(t)=v(t) i(t)'), xlabel('wt, degrees')
subplot(223), plot(wt, pr, wt, P, wt,xline), grid
title('pr(t)   Eq. 2.6'), xlabel('wt, degrees')
subplot(224), plot(wt, px, wt, xline), grid
title('px(t)  Eq. 2.8'), xlabel('wt, degrees')
subplot(111)
disp('(b) From P and Q formulas using phasor values ')
P=V*I*cos(theta)                                % Average power
Q = V*I*sin(theta)                             % Reactive power
```

The result for the inductive load $Z = 1.25\angle 60°\Omega$ is

```
    Enter voltage peak amplitude Vm = 100
    Enter voltage phase angle in degree thatav = 0
    Enter magnitude of the load impedance  Z = 1.25
    Enter load phase angle in degree gama = 60

(a) Estimate from the plots

    P =
         2000
    Q =
         3464
```

(b) For the inductive load $Z = 1.25\angle 60°\Omega$, the rms values of voltage and current are

$$V = \frac{100\angle 0°}{1.414} = 70.71\angle 0° \text{ V}$$

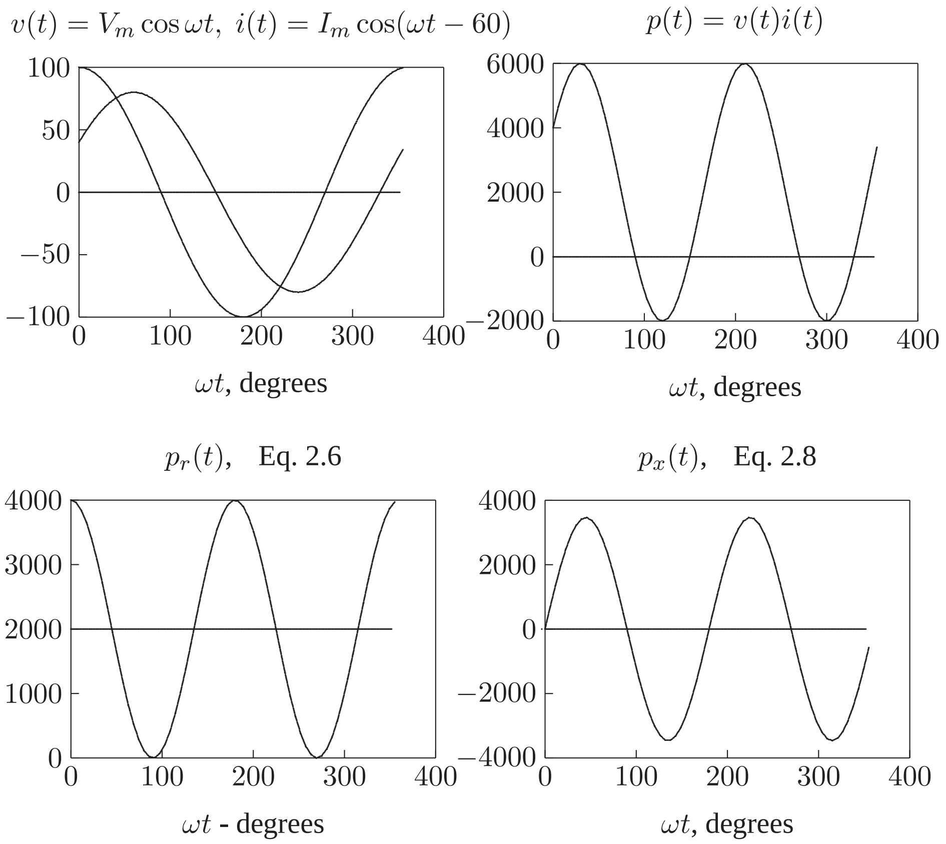


FIGURE 3
Instantaneous current, voltage, power, Eqs. 2.6 and 2.8.

$$I = \frac{70.71\angle 0^\circ}{1.25\angle 60^\circ} = 56.57\angle{-60^\circ} \ \text{ A}$$

Using (2.7) and (2.9), we have

$$P = (70.71)(56.57)\cos(60) = 2000 \ \text{ W}$$
$$Q = (70.71)(56.57)\sin(60) = 3464 \ \text{ Var}$$

Running the above program for the capacitive load $Z = 2.0\angle{-30^\circ}\ \Omega$ will result in

```
(a) Estimate from the plots

P =
    2165
Q =
   -1250
```

Similarly, for $Z = 2.5\angle 0^\circ$ Ω, we get

```
   P =
          2000
Q =
          0
```

(c) With the above three loads connected in parallel across the supply, the total real and reactive powers are

$$P = 2000 + 2165 + 2000 = 6165 \text{ W}$$
$$Q = 3464 - 1250 + 0 = 2214 \text{ Var}$$

2.2. A single-phase load is supplied with a sinusoidal voltage

$$v(t) = 200\cos(377t)$$

The resulting instantaneous power is

$$p(t) = 800 + 1000\cos(754t - 36.87^\circ)$$

(a) Find the complex power supplied to the load.
(b) Find the instantaneous current $i(t)$ and the rms value of the current supplied to the load.
(c) Find the load impedance.
(d) Use *MATLAB* to plot $v(t)$, $p(t)$, and $i(t) = p(t)/v(t)$ over a range of 0 to 16.67 ms in steps of 0.1 ms. From the current plot, estimate the peak amplitude, phase angle and the angular frequency of the current, and verify the results obtained in part (b). Note in *MATLAB* the command for array or element-by-element division is `./`.

$$\begin{aligned} p(t) &= 800 + 1000\cos(754t - 36.87^\circ) \\ &= 800 + 1000\cos 36.87^\circ \cos 754t + \sin 36.87^\circ \sin 754t \\ &= 800 + 800\cos 754t + 600\sin 754t \\ &= 800[1 + \cos 2(377)t] + 600\sin 2(377)t \end{aligned}$$

$p(t)$ is in the same form as (2.5), thus $P = 600$ W, and $Q = 600$, Var, or

$$S = 800 + j600 = 1000\angle 36.87^\circ \text{ VA}$$

(b) Using $S = \frac{1}{2}V_m {I_m}^*$, we have

$$1000\angle 36.87^\circ = \frac{1}{2}200\angle 0^\circ I_m$$

or

$$I_m = 10\angle{-36.87^\circ} \ \text{A}$$

Therefore, the instantaneous current is

$$i(t) = 10cos(377t - 36.87^\circ) \ \text{A}$$

(c)

$$Z_L = \frac{V}{I} = \frac{200\angle 0^\circ}{10\angle{-36.87^\circ}} = 20\angle 36.87^\circ \ \ \Omega$$

(d) We use the following command

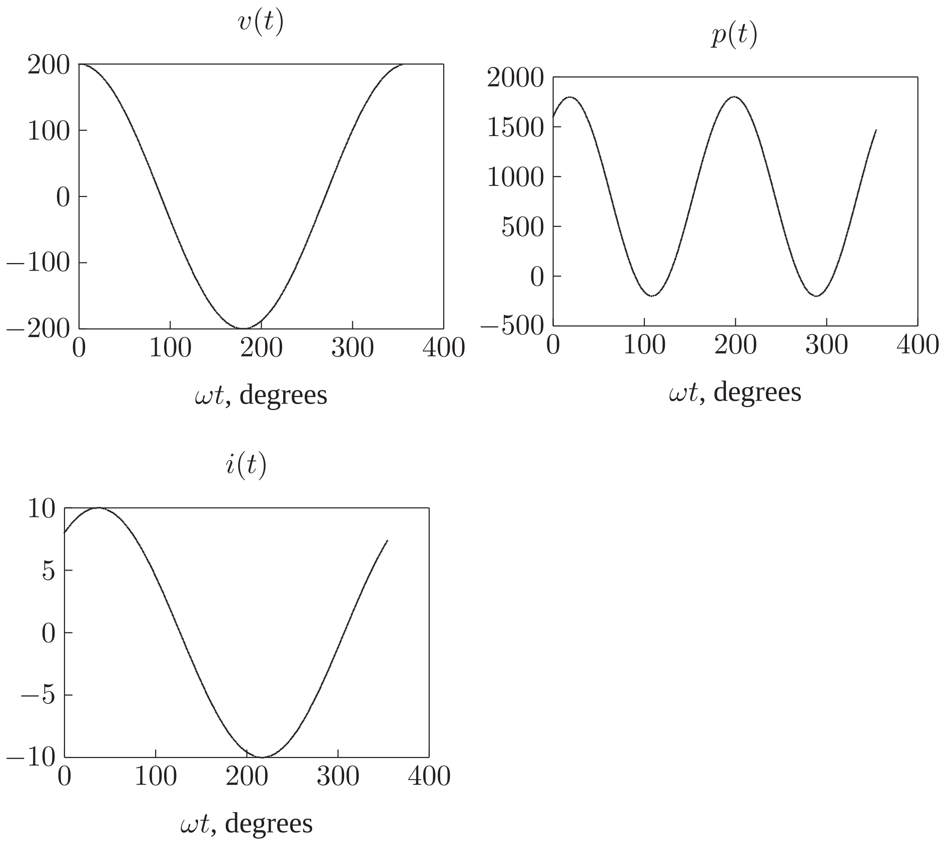


FIGURE 4
Instantaneous voltage, power, and current for Problem 2.2.

```
Vm = 200;
t=0:.0001:0.01667;                          % wt from 0 to 2*pi
v=Vm*cos(377*t);                        % Instantaneous voltage
p = 800 + 1000*cos(754*t - 36.87*pi/180);% Instantaneous power
i=p./v;                                 % Instantaneous current
wt=180/pi*377*t;                  % converting radian to degree
xline = zeros(1, length(wt));       % generates a zero vector
subplot(221), plot(wt, v, wt, xline), grid
xlabel('wt,  degrees'), title('v(t)')
subplot(222), plot(wt, p, wt, xline), grid
xlabel('wt,  degrees'), title('p(t)')
subplot(223), plot(wt, i, wt, xline), grid
xlabel('wt, degrees'), title('i(t)'), subplot(111)
```

The result is shown in Figure 4. The inspection of current plot shows that the peak amplitude of the current is 10 A, lagging voltage by $36.87°$, with an angular frequency of 377 Rad/sec.

2.3. An inductive load consisting of R and X in series feeding from a 2400-V rms supply absorbs 288 kW at a lagging power factor of 0.8. Determine R and X.

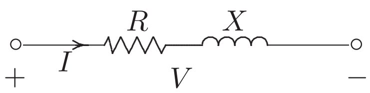


FIGURE 5
An inductive load, with R and X in series.

$$\theta = \cos^{-1} 0.8 = 36.87°$$

The complex power is

$$S = \frac{288}{0.8}\angle 36.87° = 360\angle 36.87° \text{ kVA}$$

The current given from $S = VI^*$, is

$$I = \frac{360 \times 10^3\angle -36.87°}{2400\angle 0°} = 150\angle -36.87 \text{ A}$$

Therefore, the series impedance is

$$Z = R + jX = \frac{V}{I} = \frac{2400\angle 0°}{150\angle -36.87°} = 12.8 + j9.6 \ \Omega$$

Therefore, $R = 12.8\ \Omega$ and $X = 9.6\ \Omega$.

2.4. An inductive load consisting of R and X in parallel feeding from a 2400-V rms supply absorbs 288 kW at a lagging power factor of 0.8. Determine R and X.

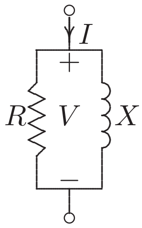


FIGURE 6
An inductive load, with R and X in parallel.

The complex power is

$$S = \frac{288}{0.8}\angle 36.87^\circ = 360\angle 36.87^\circ \ \text{ kVA}$$
$$= 288 \text{ kW} + j216 \text{ kvar}$$

$$R = \frac{|V|^2}{P} = \frac{(2400)^2}{288 \times 10^3} = 20 \ \ \Omega$$
$$X = \frac{|V|^2}{Q} = \frac{(2400)^2}{216 \times 10^3} = 26.667 \ \ \Omega$$

2.5. Two loads connected in parallel are supplied from a single-phase 240-V rms source. The two loads draw a total real power of 400 kW at a power factor of 0.8 lagging. One of the loads draws 120 kW at a power factor of 0.96 leading. Find the complex power of the other load.

$$\theta = \cos^{-1} 0.8 = 36.87^\circ$$

The total complex load is

$$S = \frac{400}{0.8}\angle 36.87^\circ = 500\angle 36.87^\circ \ \text{ kVA}$$
$$= 400 \text{ kW} + j300 \text{ kvar}$$

The 120 kW load complex power is

$$S = \frac{120}{0.96}\angle -16.26^\circ = 125\angle -16.26^\circ \ \text{ kVA}$$
$$= 120 \text{ kW} - j35 \text{ kvar}$$

Therefore, the second load complex power is

$$S_2 = 400 + j300 - (120 - j35) = 280 \text{ kW} + j335 \text{ kvar}$$

2.6. The load shown in Figure 7 consists of a resistance R in parallel with a capacitor of reactance X. The load is fed from a single-phase supply through a line of impedance $8.4 + j11.2\ \Omega$. The rms voltage at the load terminal is $1200\angle 0°$ V rms, and the load is taking 30 kVA at 0.8 power factor leading.
(a) Find the values of R and X.
(b) Determine the supply voltage V.

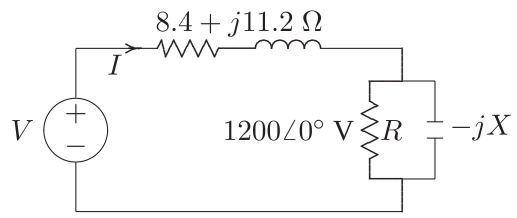


FIGURE 7
Circuit for Problem 2.6.

$$\theta = \cos^{-1} 0.8 = 36.87°$$

The complex power is

$$S = 30\angle{-36.87°} = 24 \text{ kW} - j18 \text{ kvar}$$

(a)

$$R = \frac{|V|^2}{P} = \frac{(1200)^2}{24000} = 60\ \ \Omega$$
$$X = \frac{|V|^2}{Q} = \frac{(1200)^2}{18000} = 80\ \ \Omega$$

From $S = VI^*$, the current is

$$I = \frac{30000\angle 36.87°}{1200\angle 0°} = 25\angle 36.87\ \ \text{A}$$

Thus, the supply voltage is

$$\begin{aligned} V &= 1200\angle 0° + 25\angle 36.87°(8.4 + j11.2) \\ &= 1200 + j350 = 1250\angle 16.26°\ \ \text{V} \end{aligned}$$

2.7. Two impedances, $Z_1 = 0.8 + j5.6\ \Omega$ and $Z_2 = 8 - j16\ \Omega$, and a single-phase motor are connected in parallel across a 200-V rms, 60-Hz supply as shown in Figure 8. The motor draws 5 kVA at 0.8 power factor lagging.

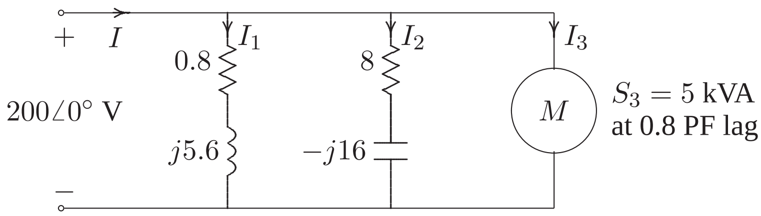


FIGURE 8
Circuit for Problem 2.7.

(a) Find the complex powers S_1, S_2 for the two impedances, and S_3 for the motor.
(b) Determine the total power taken from the supply, the supply current, and the overall power factor.
(c) A capacitor is connected in parallel with the loads. Find the kvar and the capacitance in μF to improve the overall power factor to unity. What is the new line current?

(a) The load complex power are

$$S_1 = \frac{|V|^2}{Z_1^*} = \frac{(200)^2}{0.8 - j5.6} = 1000 + j7000 \text{ VA}$$
$$S_2 = \frac{|V|^2}{Z_2^*} = \frac{(200)^2}{8 + j16} = 1000 - j2000 \text{ VA}$$
$$S_3 = 5000\angle 36.87° = 4000 + j3000 \text{ VA}$$

Therefore, the total complex power is

$$S_t = 6 + j8 = 10\angle 53.13° \text{ kVA}$$

(b) From $S = VI^*$, the current is

$$I = \frac{10000\angle -53.13°}{200\angle 0°} = 50\angle -53.13 \text{ A}$$

and the power factor is $\cos 53.13° = 0.6$ lagging.

(c) For overall unity power factor, $Q_C = 8000$ Var, and the capacitive impedance is

$$Z_C = \frac{|V|^2}{S_C{}^*} = \frac{(200)^2}{j8000} = -j5 \;\; \Omega$$

and the capacitance is

$$C = \frac{10^6}{(2\pi)(60)(5)} = 530.5 \;\; \mu\text{F}$$

The new current is

$$I = \frac{6000\angle 0^\circ}{200\angle 0^\circ} = 30\angle 0 \;\; \text{A}$$

2.8. Two single-phase ideal voltage sources are connected by a line of impedance of $0.7 + j2.4\ \Omega$ as shown in Figure 9. $V_1 = 500\angle 16.26^\circ$ V and $V_2 = 585\angle 0^\circ$ V. Find the complex power for each machine and determine whether they are delivering or receiving real and reactive power. Also, find the real and the reactive power loss in the line.

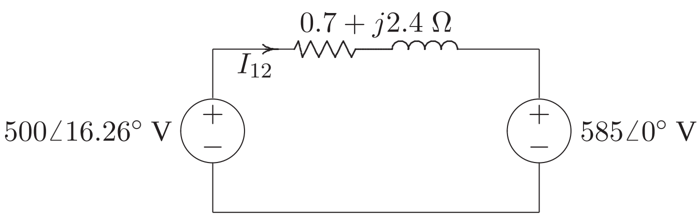


FIGURE 9
Circuit for Problem 2.8.

$$I_{12} = \frac{500\angle 16.26^\circ - 585\angle 0^\circ}{0.7 + j2.4} = 42 + j56 = 70\angle 53.13^\circ \;\; \text{A}$$

$$\begin{aligned}
S_{12} = V_1 I_{12}^* = (500\angle 16.26^\circ)(70\angle{-53.13^\circ}) &= 35000\angle{-36.87^\circ} \\
&= 28000 - j21000 \;\; \text{VA} \\
S_{21} = V_2 I_{21}^* = (585\angle 0^\circ)(-70\angle{-53.13^\circ}) &= 40950\angle{-53.13^\circ} \\
&= -24570 + j32760 \;\; \text{VA}
\end{aligned}$$

From the above results, since P_1 is positive and P_2 is negative, source 1 generates 28 kW, and source 2 receives 24.57 kW, and the real power loss is 3.43 kW. Similarly, since Q_1 is negative, source 1 receives 21 kvar and source 2 delivers 32.76 kvar. The reactive power loss in the line is 11.76 kvar.

2.9. Write a *MATLAB* program for the system of Example 2.5 such that the voltage magnitude of source 1 is changed from 75 percent to 100 percent of the given value in steps of 1 volt. The voltage magnitude of source 2 and the phase angles of the two sources is to be kept constant. Compute the complex power for each source and the line loss. Tabulate the reactive powers and plot Q_1, Q_2, and Q_L versus voltage magnitude $|V_1|$. From the results, show that the flow of reactive power along the interconnection is determined by the magnitude difference of the terminal voltages.

We use the following commands

```
E1 = input('Source # 1 Voltage Mag. = ');
a1 = input('Source # 1 Phase Angle  = ');
E2 = input('Source # 2 Voltage Mag. = ');
a2 = input('Source # 2 Phase Angle  = ');
R = input('Line Resistance = ');
X = input('Line Reactance = ');
Z = R + j*X;                                % Line impedance
E1 = (0.75*E1:1:E1)';      % Change E1 form 75% to 100% E1
a1r = a1*pi/180;                 % Convert degree to radian
k = length(E1);
E2 = ones(k,1)*E2;%create col. Array of same length for E2
a2r = a2*pi/180;                 % Convert degree to radian
V1=E1.*cos(a1r) + j*E1.*sin(a1r);
V2=E2.*cos(a2r) + j*E2.*sin(a2r);
I12 = (V1 - V2)./Z;  I21=-I12;
S1= V1.*conj(I12);  P1 = real(S1);  Q1 = imag(S1);
S2= V2.*conj(I21);  P2 = real(S2);  Q2 = imag(S2);
SL= S1+S2;        PL = real(SL);  QL = imag(SL);
Result1=[E1, Q1,  Q2, QL];
disp('     E1       Q-1       Q-2       Q-L ')
disp(Result1)
plot(E1, Q1,  E1, Q2,  E1, QL), grid
xlabel(' Source #1 Voltage Magnitude')
ylabel(' Q, var')
text(112.5, -180, 'Q2')
text(112.5,  5,'QL'), text(112.5, 197, 'Q1')
```

The result is

```
Source # 1 Voltage Mag. = 120
Source # 1 Phase Angle  = -5
Source # 2 Voltage Mag. = 100
Source # 2 Phase Angle  = 0
Line Resistance = 1
Line Reactance = 7

E1        Q-1        Q-2        Q-L

 90.0000 -105.5173  129.1066   23.5894
 91.0000  -93.9497  114.9856   21.0359
 92.0000  -82.1021  100.8646   18.7625
 93.0000  -69.9745   86.7435   16.7690
 94.0000  -57.5669   72.6225   15.0556
 95.0000  -44.8794   58.5015   13.6221
 96.0000  -31.9118   44.3804   12.4687
 97.0000  -18.6642   30.2594   11.5952
 98.0000   -5.1366   16.1383   11.0017
 99.0000    8.6710    2.0173   10.6883
100.0000   22.7586  -12.1037   10.6548
101.0000   37.1262  -26.2248   10.9014
102.0000   51.7737  -40.3458   11.4279
103.0000   66.7013  -54.4668   12.2345
104.0000   81.9089  -68.5879   13.3210
105.0000   97.3965  -82.7089   14.6876
106.0000  113.1641  -96.8299   16.3341
107.0000  129.2117 -110.9510   18.2607
108.0000  145.5393 -125.0720   20.4672
109.0000  162.1468 -139.1931   22.9538
110.0000  179.0344 -153.3141   25.7203
111.0000  196.2020 -167.4351   28.7669
112.0000  213.6496 -181.5562   32.0934
113.0000  231.3772 -195.6772   35.7000
114.0000  249.3848 -209.7982   39.5865
115.0000  267.6724 -223.9193   43.7531
116.0000  286.2399 -238.0403   48.1996
117.0000  305.0875 -252.1614   52.9262
118.0000  324.2151 -266.2824   57.9327
119.0000  343.6227 -280.4034   63.2193
120.0000  363.3103 -294.5245   68.7858
```

Examination of Figure 10 shows that the flow of reactive power along the interconnection is determined by the voltage magnitude difference of terminal voltages.

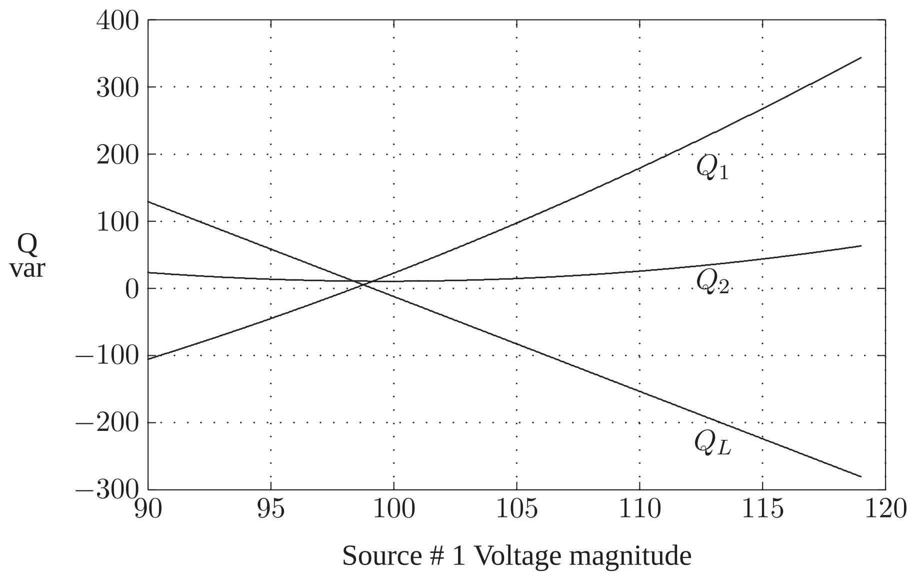


FIGURE 10
Reactive power versus voltage magnitude.

2.10. A balanced three-phase source with the following instantaneous phase voltages

$$v_{an} = 2500\cos(\omega t)$$
$$v_{bn} = 2500\cos(\omega t - 120°)$$
$$v_{cn} = 2500\cos(\omega t - 240°)$$

supplies a balanced Y-connected load of impedance $Z = 250\angle 36.87°\ \Omega$ per phase.
(a) Using *MATLAB*, plot the instantaneous powers p_a, p_b, p_c and their sum versus ωt over a range of $0 : 0.05 : 2\pi$ on the same graph. Comment on the nature of the instantaneous power in each phase and the total three-phase real power.
(b) Use (2.44) to verify the total power obtained in part (a).

We use the following commands

```
wt=0:.02:2*pi;
pa=25000*cos(wt).*cos(wt-36.87*pi/180);
pb=25000*cos(wt-120*pi/180).*cos(wt-120*pi/180-36.87*pi/180);
pc=25000*cos(wt-240*pi/180).*cos(wt-240*pi/180-36.87*pi/180);
p = pa+pb+pc;
```

```
plot(wt, pa, wt, pb, wt, pc, wt, p), grid
xlabel('Radian')
disp('(b)')
V = 2500/sqrt(2);
gama = acos(0.8);
Z = 250*(cos(gama)+j*sin(gama));
I = V/Z;
P = 3*V*abs(I)*0.8
```

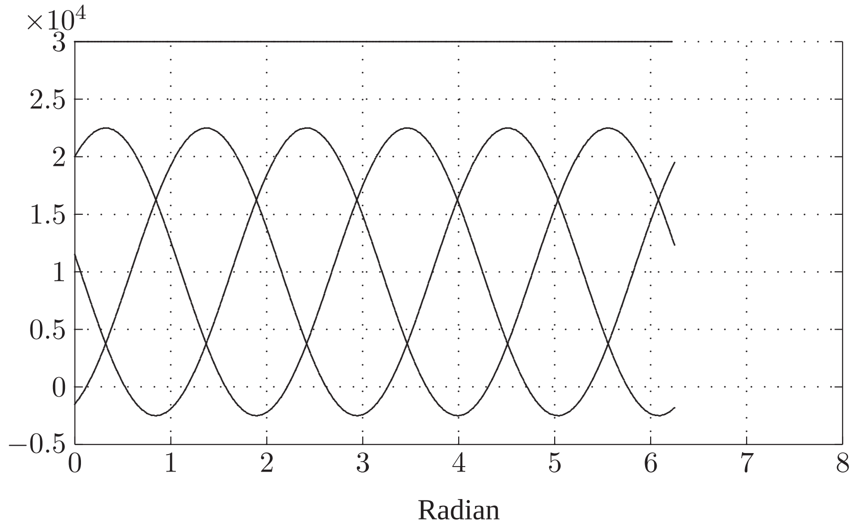


FIGURE 11
Instantaneous powers and their sum for Problem 2.10.

(b)

$$V = \frac{2500}{\sqrt{2}} = 1767.77\angle 0^\circ \;\; \text{V}$$
$$I = \frac{1767.77\angle 0^\circ}{250\angle 36.87^\circ} = 7.071\angle{-36.87^\circ} \;\; \text{A}$$
$$P = (3)(1767.77)(7.071)(0.8) = 30000 \;\; \text{W}$$

2.11. A 4157-V rms three-phase supply is applied to a balanced Y-connected three-phase load consisting of three identical impedances of $48\angle 36.87^\circ\ \Omega$. Taking the phase to neutral voltage V_{an} as reference, calculate
(a) The phasor currents in each line.
(b) The total active and reactive power supplied to the load.

$$V_{an} = \frac{4157}{\sqrt{3}} = 2400 \;\; \text{V}$$

With V_{an} as reference, the phase voltages are:

$$V_{an} = 2400\angle 0^\circ \text{ V} \qquad V_{bn} = 2400\angle -120^\circ \text{ V} \qquad V_{an} = 2400\angle -240^\circ \text{ V}$$

(a) The phasor currents are:

$$I_a = \frac{V_{an}}{Z} = \frac{2400\angle 0^\circ}{48\angle 36.87^\circ} = 50\angle -36.87^\circ \text{ A}$$
$$I_b = \frac{V_{bn}}{Z} = \frac{2400\angle -120^\circ}{48\angle 36.87^\circ} = 50\angle -156.87^\circ \text{ A}$$
$$I_c = \frac{V_{cn}}{Z} = \frac{2400\angle -240^\circ}{48\angle 36.87^\circ} = 50\angle -276.87^\circ \text{ A}$$

(b) The total complex power is

$$S = 3V_{an}I_a^* = (3)(2400\angle 0^\circ)(50\angle 36.87^\circ) = 360\angle 36.87^\circ \text{ kVA}$$
$$= 288 \text{ kW} + j216 \text{ KVAR}$$

2.12. Repeat Problem 2.11 with the same three-phase impedances arranged in a Δ connection. Take V_{ab} as reference.

$$V_{an} = \frac{4157}{\sqrt{3}} = 2400 \text{ V}$$

With V_{ab} as reference, the phase voltages are:

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{4157\angle 0^\circ}{48\angle 36.87^\circ} = 86.6\angle -36.87^\circ \text{ A}$$

$$I_a = \sqrt{3}\angle -30^\circ I_{ab} = (\sqrt{3}\angle -30^\circ)(86.6\angle -36.87^\circ = 150\angle -66.87^\circ \text{ A}$$

For positive phase sequence, current in other lines are

$$I_b = 150\angle -186.87^\circ \text{ A}, \text{ and } I_c = 150\angle 53.13^\circ \text{ A}$$

(b) The total complex power is

$$S = 3V_{ab}I_{ab}^* = (3)(4157\angle 0^\circ)(86.6\angle 36.87^\circ) = 1080\angle 36.87^\circ \text{ kVA}$$
$$= 864 \text{ kW} + j648 \text{ kvar}$$

2.13. A balanced delta connected load of $15 + j18\ \Omega$ per phase is connected at the end of a three-phase line as shown in Figure 12. The line impedance is $1 + j2\ \Omega$ per phase. The line is supplied from a three-phase source with a line-to-line voltage of 207.85 V rms. Taking V_{an} as reference, determine the following:

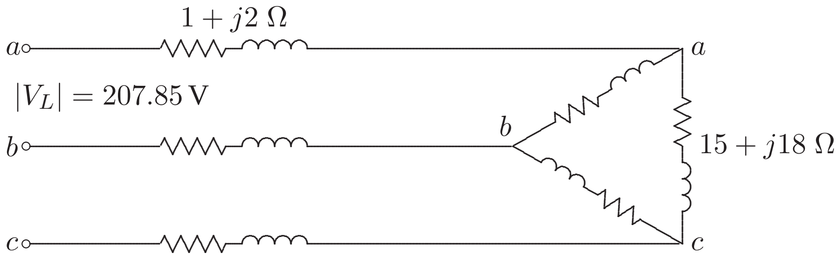


FIGURE 12
Circuit for Problem 2.13.

(a) Current in phase a.
(b) Total complex power supplied from the source.
(c) Magnitude of the line-to-line voltage at the load terminal.

$$V_{an} = \frac{207.85}{\sqrt{3}} = 120 \;\; \text{V}$$

Transforming the delta connected load to an equivalent Y-connected load, result in the phase 'a' equivalent circuit, shown in Figure 13.

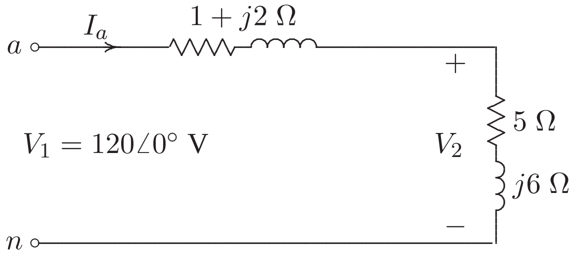


FIGURE 13
The per phase equivalent circuit for Problem 2.13.

(a)

$$I_a = \frac{120\angle 0^\circ}{6 + j8} = 12\angle{-53.13^\circ} \;\; \text{A}$$

(b) The total complex power is

$$\begin{aligned} S = 3V_{an}I_a^* = (3)(120\angle 0^\circ)(12\angle 53.13^\circ &= 4320\angle 53.13^\circ \;\; \text{VA} \\ &= 2592 \text{ W} + j3456 \text{ Var} \end{aligned}$$

(c)

$$V_2 = 120\angle 0^\circ - (1 + j2)(12\angle{-53.13^\circ} = 93.72\angle{-2.93^\circ} \;\; \text{A}$$

Thus, the magnitude of the line-to-line voltage at the load terminal is $V_L = \sqrt{3}(93.72) = 162.3$ V.

2.14. Three parallel three-phase loads are supplied from a 207.85-V rms, 60-Hz three-phase supply. The loads are as follows:

Load 1: A 15 HP motor operating at full-load, 93.25 percent efficiency, and 0.6 lagging power factor.
Load 2: A balanced resistive load that draws a total of 6 kW.
Load 3: A Y-connected capacitor bank with a total rating of 16 kvar.

(a) What is the total system kW, kvar, power factor, and the supply current per phase?
(b) What is the system power factor and the supply current per phase when the resistive load and induction motor are operating but the capacitor bank is switched off?

The real power input to the motor is

$$\begin{aligned} P_1 &= \frac{(15)(746)}{0.9325} = 12 \text{ kW} \\ S_1 &= \frac{12}{0.6}\angle 53.13^\circ \text{ kVA} = 12 \text{ kW} + j16 \text{ kvar} \\ S_2 &= 6 \text{ kW} + j0 \text{ kvar} \\ S_3 &= 0 \text{ kW} - j16 \text{ kvar} \end{aligned}$$

(a) The total complex power is

$$S = 18\angle 0^\circ \text{ kVA} = 18 \text{ kW} + j0 \text{ kvar}$$

The supply current is

$$I = \frac{18000}{(3)(120)} = 50\angle 0^\circ \text{ A}, \qquad \text{at unity power factor}$$

(b) With the capacitor switched off, the total power is

$$S = 18 + j16 = 24.08\angle 41.63^\circ \text{ kVA}$$

$$I = \frac{24083\angle -41.63}{(3)(120\angle 0^\circ)} = 66.89\angle -41.63^\circ \text{ A}$$

The power factor is $\cos 41.63^\circ = 0.747$ lagging.

2.15. Three loads are connected in parallel across a 12.47 kV three-phase supply.

Load 1: Inductive load, 60 kW and 660 kvar.
Load 2: Capacitive load, 240 kW at 0.8 power factor.
Load 3: Resistive load of 60 kW.

(a) Find the total complex power, power factor, and the supply current.
(b) A Y-connected capacitor bank is connected in parallel with the loads. Find the total kvar and the capacitance per phase in μF to improve the overall power factor to 0.8 lagging. What is the new line current?

$$\begin{aligned} S_1 &= \;\;60 \text{ kW} + j660 \text{ kvar} \\ S_2 &= 240 \text{ kW} - j180 \text{ kvar} \\ S_3 &= \;\;60 \text{ kW} + \;\;\; j0 \text{ kvar} \end{aligned}$$

(a) The total complex power is

$$S = 360 \text{ kW} + j480 \text{ kvar} = 600\angle 53.13^\circ \text{ kVA}$$

The phase voltage is

$$V = \frac{12.47}{\sqrt{3}} = 7.2\angle 0^\circ \;\; \text{kV}$$

The supply current is

$$I = \frac{600\angle{-53.13^\circ}}{(3)(7.2)} = 27.77\angle{-53.13^\circ} \;\; \text{A}$$

The power factor is $\cos 53.13^\circ = 0.6$ lagging.

(b) The net reactive power for 0.8 power factor lagging is

$$Q' = 360 \tan 36.87^\circ = 270 \;\; \text{kvar}$$

Therefore, the capacitor kvar is $Q_c = 480 - 270 = 210$ kvar, or $S_c = -j210$ kVA.

$$X_c = \frac{|V_L|^2}{S_c^*} = \frac{(12.47 \times 1000)^2}{j210000} = -j740.48 \; \Omega$$

$$C = \frac{10^6}{(2\pi)(60)(740.48)} = 3.58\mu\text{F}$$

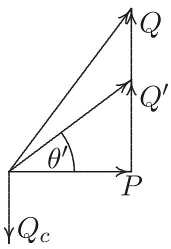


FIGURE 14
The power diagram for Problem 2.15.

$$I = \frac{S^*}{V^*} = \frac{360 - j270}{(3)(7.2)} = 20.835\angle{-36.87^\circ} \text{ A}$$

2.16. A balanced Δ-connected load consisting of a pure resistances of 18 Ω per phase is in parallel with a purely resistive balanced Y-connected load of 12 Ω per phase as shown in Figure 15. The combination is connected to a three-phase balanced supply of 346.41-V rms (line-to-line) via a three-phase line having an inductive reactance of $j3$ Ω per phase. Taking the phase voltage V_{an} as reference, determine
(a) The current, real power, and reactive power drawn from the supply.
(b) The line-to-neutral and the line-to-line voltage of phase a at the combined load terminals.

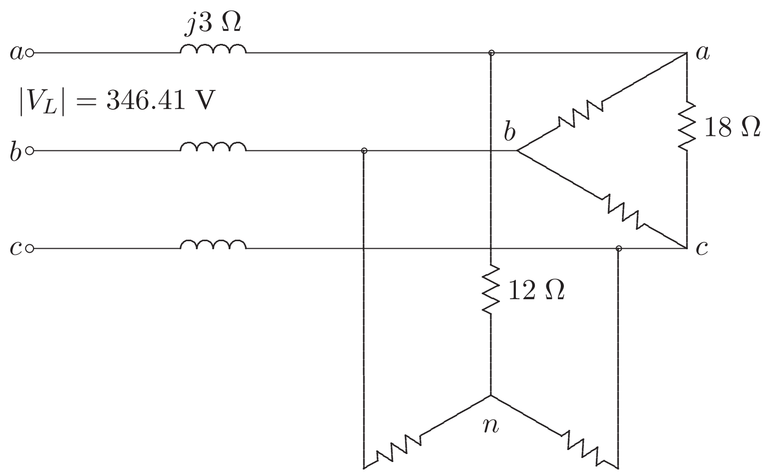


FIGURE 15
Circuit for Problem 2.16.

Transforming the delta connected load to an equivalent Y-connected load, result in the phase 'a' equivalent circuit, shown in Figure 16.

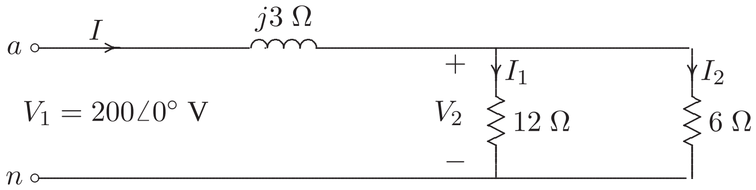


FIGURE 16
The per phase equivalent circuit for Problem 2.16.

(a)

$$V_{an} = \frac{346.41}{\sqrt{3}} = 200 \text{ V}$$

The input impedance is

$$Z = \frac{(12)(6)}{12+6} + j3 = 4 + j3 \ \ \Omega$$

$$I_a = \frac{200\angle 0^\circ}{4+j3} = 40\angle{-36.87^\circ} \ \ \text{A}$$

The total complex power is

$$\begin{aligned} S = 3V_{an}I_a^* = (3)(200\angle 0^\circ)(40\angle 36.87^\circ) &= 24000\angle 36.87^\circ \text{ VA} \\ &= 19200 \text{ W} + j14400 \text{ Var} \end{aligned}$$

(b)

$$V_2 = 200\angle 0^\circ - (j3)(40\angle{-36.87^\circ}) = 160\angle{-36.87^\circ} \ \ \text{A}$$

Thus, the magnitude of the line-to-line voltage at the load terminal is $V_L = \sqrt{3}(160) =$ 277.1 V.

CHAPTER 3 PROBLEMS

3.1. A three-phase, 318.75-kVA, 2300-V alternator has an armature resistance of 0.35 Ω/phase and a synchronous reactance of 1.2 Ω/phase. Determine the no-load line-to-line generated voltage and the voltage regulation at
(a) Full-load kVA, 0.8 power factor lagging, and rated voltage.
(b) Full-load kVA, 0.6 power factor leading, and rated voltage.

$$V_\phi = \frac{2300}{\sqrt{3}} = 1327.9 \text{ V}$$

(a) For 318.75 kVA, 0.8 power factor lagging, $S = 318750\angle 36.87°$ VA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{318750\angle{-36.87°}}{(3)(1327.9)} = 80\angle{-36.87°} \text{ A}$$

$$E_\phi = 1327.9 + (0.35 + j1.2)(80\angle{-36.87°}) = 1409.2\angle 2.44° \text{ V}$$

The magnitude of the no-load generated voltage is
$E_{LL} = \sqrt{3}\ 1409.2 = 2440.8$ V, and the voltage regulation is

$$V.R. = \frac{2440.8 - 2300}{2300} \times 100 = 6.12\%$$

(b) For 318.75 kVA, 0.6 power factor leading, $S = 318750\angle{-53.13°}$ VA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{318750\angle 53.13°}{(3)(1327.9} = 80\angle 53.13° \text{ A}$$

$$E_\phi = 1327.9 + (0.35 + j1.2)(80\angle 53.13^\circ) = 1270.4\angle 3.61^\circ \;\; \text{V}$$

The magnitude of the no-load generated voltage is
$E_{LL} = \sqrt{3}\ 1270.4 = 2220.4$ V, and the voltage regulation is

$$V.R. = \frac{2200.4 - 2300}{2300} \times 100 = -4.33\%$$

3.2. A 60-MVA, 69.3-kV, three-phase synchronous generator has a synchronous reactance of 15 Ω/phase and negligible armature resistance.

(a) The generator is delivering rated power at 0.8 power factor lagging at the rated terminal voltage to an infinite bus bar. Determine the magnitude of the generated emf per phase and the power angle δ.

(b) If the generated emf is 36 kV per phase, what is the maximum three-phase power that the generator can deliver before losing its synchronism?

(c) The generator is delivering 48 MW to the bus bar at the rated voltage with its field current adjusted for a generated emf of 46 kV per phase. Determine the armature current and the power factor. State whether power factor is lagging or leading?

$$V_\phi = \frac{69.3}{\sqrt{3}} = 40 \;\; \text{kV}$$

(a) For 60 kVA, 0.8 power factor lagging, $S = 60000\angle 36.87^\circ$ kVA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{60000\angle -36.87^\circ}{(3)(40)} = 500\angle -36.87^\circ \;\; \text{A}$$

$$E_\phi = 40 + (j15)(500\angle -36.87^\circ) \times 10^{-3} = 44.9\angle 7.675^\circ \;\; \text{kV}$$

(b)

$$P_{max} = \frac{3|E||V|}{X_s} = \frac{(3)(36)(40)}{15} = 288 \;\; \text{MW}$$

(c) For $P = 48$ MW, and $E = 46$ KV/phase, the power angle is given by

$$48 = \frac{(3)(46)(40)}{15} \sin\delta$$

or

$$\delta = 7.4947^\circ$$

and solving for the armature current from $E = V + jX_sI_a$, we have

$$I_a = \frac{46000\angle 7.4947^\circ - 40000\angle 0^\circ}{j15} = 547.47\angle -43.06^\circ \ \ \text{A}$$

The power factor is $\cos^{-1} 43.06 = 0.7306$ lagging.

3.3. A 24,000-kVA, 17.32-kV, Y-connected synchronous generator has a synchronous reactance of 5 Ω/phase and negligible armature resistance.

(a) At a certain excitation, the generator delivers rated load, 0.8 power factor lagging to an infinite bus bar at a line-to-line voltage of 17.32 kV. Determine the excitation voltage per phase.

(b) The excitation voltage is maintained at 13.4 KV/phase and the terminal voltage at 10 KV/phase. What is the maximum three-phase real power that the generator can develop before pulling out of synchronism?

(c) Determine the armature current for the condition of part (b).

$$V_\phi = \frac{17.32}{\sqrt{3}} = 10 \ \ \text{kV}$$

(a) For 24000 kVA, 0.8 power factor lagging, $S = 24000\angle 36.87^\circ$ kVA.

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{24000\angle -36.87^\circ}{(3)(10)} = 800\angle -36.87^\circ \ \ \text{A}$$

$$E_\phi = 10 + (j5)(800\angle -36.87^\circ) \times 10^{-3} = 12.806\angle 14.47^\circ \ \ \text{kV}$$

(b)

$$P_{max} = \frac{3|E||V|}{X_s} = \frac{(3)(13.4)(10)}{5} = 80.4 \ \ \text{MW}$$

(c) At maximum power transfer $\delta = 90^\circ$, and solving for the armature current from $E = V + jX_sI_a$, we have

$$I_a = \frac{13400\angle 90^\circ - 10000\angle 0^\circ}{j5} = 3344\angle 36.73^\circ \ \ \text{A}$$

The power factor is $\cos^{-1} 36.73 = 0.7306$ leading.

3.4. A 34.64-kV, 60-MVA, three-phase salient-pole synchronous generator has a direct axis reactance of 13.5 Ω and a quadrature-axis reactance of 9.333 Ω. The armature resistance is negligible.

(a) Referring to the phasor diagram of a salient-pole generator shown in Figure 17, show that the power angle δ is given by

$$\delta = \tan^{-1}\left(\frac{X_q|I_a|\cos\theta}{V + X_q|I_a|\sin\theta}\right)$$

(b) Compute the load angle δ and the per phase excitation voltage E when the generator delivers rated MVA, 0.8 power factor lagging to an infinite bus bar of 34.64-kV line-to-line voltage.

(c) The generator excitation voltage is kept constant at the value found in part (b). Use *MATLAB* to obtain a plot of the power angle curve, i.e., equation (3.26) over a range of $d = 0 : .05 : 180°$. Use the command $[P_{max}, k]$ = max(P); $d_{max} = d(k)$, to obtain the steady-state maximum power P_{max} and the corresponding power angle d_{max}.

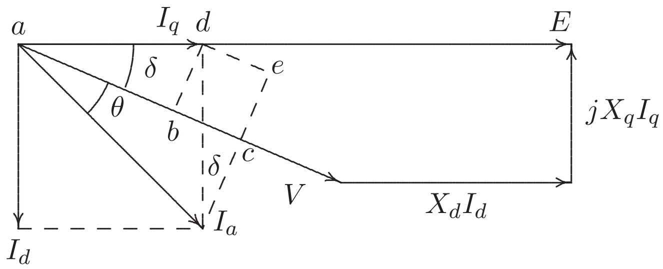


FIGURE 17
Phasor diagram of a salient-pole generator for Problem 3.4.

(a) Form the phasor diagram shown in Figure 17, we have

$$\begin{aligned} |V|\sin\delta &= X_qI_q \\ &= X_q|I_a|\cos(\theta+\delta) \\ &= X_q|I_a|(\cos\theta\cos\delta - \sin\theta\sin\delta) \end{aligned}$$

or

$$\frac{\sin\delta}{\cos\delta} = \frac{X_q|I_a|\cos\theta}{V + X_q|I_a|\sin\theta}$$

or

$$\delta = \tan^{-1} \frac{X_q|I_a|\cos\theta}{V + X_q|I_a|\sin\theta}$$

(b)

$$V_\phi = \frac{34.64}{\sqrt{3}} = 20 \text{ kV}$$

(a) For 60 MVA, 0.8 power factor lagging, $S = 60\angle 36.87^\circ$ MVA

$$I_a = \frac{S^*}{3V_\phi^*} = \frac{60000\angle{-36.87^\circ}}{(3)(20)} = 1000\angle{-36.87^\circ} \text{ A}$$

$$\delta = \tan^{-1} \frac{(9.333)(1000)(0.8)}{20000 + (9.333)(1000)(0.6)} = 16.26^\circ$$

The magnitude of the no-load generated emf per phase is given by

$$\begin{aligned} |E| &= |V|\cos\delta + X_d|I_a|\sin(\theta + \delta) \\ &= 20\cos 16.26^\circ + (13.5)(1000)(10^{-3})\sin 53.13^\circ = 30 \text{ kV} \end{aligned}$$

(c) We use the following commands

```
V = 20000; Xd = 13.5; Xq = 9.333;
theta=acos(0.8);
Ia = 20E06/20000;
delta = atan(Xq*Ia*cos(theta)/(V + Xq*Ia*sin(theta)));
deltadg=delta*180/pi;
E = V*cos(delta)+Xd*Ia*sin(theta+delta);
E_KV = E/1000;  % Excitaiton voltage in kV
fprintf('Power angle = %g Degree \n', deltadg)
fprintf('E = %g kV \n\n', E_KV)
deltadg = (0:.25:180)';
delta=deltadg*pi/180;
P=3*E*V/Xd*sin(delta)+3*V^2*(Xd-Xq)/(2*Xd*Xq)*sin(2*delta);
P = P/1000000;      % Power in MW
plot(deltadg, P), grid
xlabel('Delta - Degree'), ylabel('P - MW')
[Pmax, k]=max(P); delmax=deltadg(k);
fprintf('Max power = %g MW',Pmax)
fprintf(' at power angle %g degree \n', delmax)
```

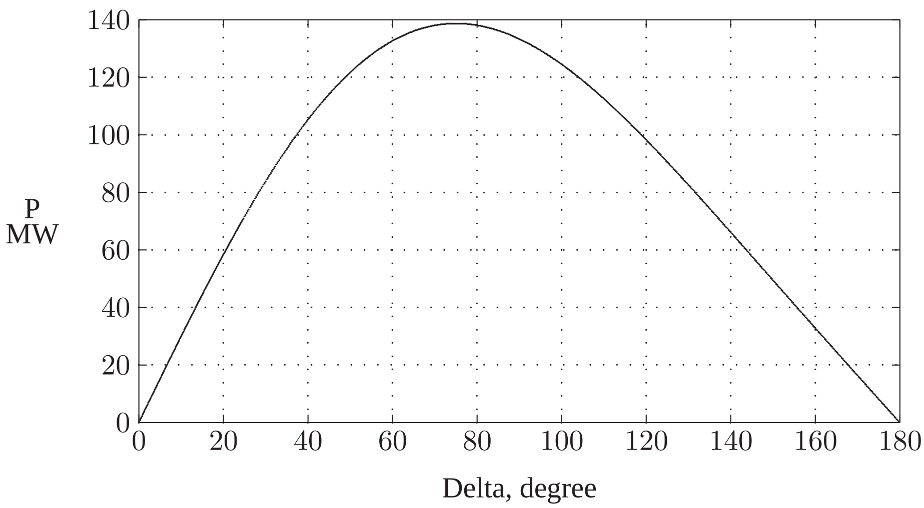


FIGURE 18
Power angle curve for Problem 3.4.

The result is

```
Power angle = 16.2598 Degree
E = 30 kV
Max power = 138.712 MW at power angle 75 degree
```

3.5. A 150-kVA, 2400/240-V single-phase transformer has the parameters as shown in Figure 19.

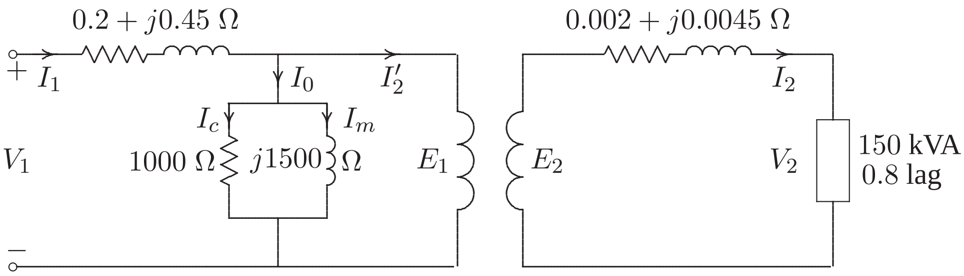


FIGURE 19
Transformer circuit for Problem 3.5.

(a) Determine the equivalent circuit referred to the high-voltage side.
(b) Find the primary voltage and voltage regulation when transformer is operating at full load 0.8 power factor lagging and 240 V.

(c) Find the primary voltage and voltage regulation when the transformer is operating at full-load 0.8 power factor leading.
(d) Verify your answers by running the **trans** program in *MATLAB* and obtain the transformer efficiency curve.

(a) Referring the secondary impedance to the primary side, the transformer equivalent impedance referred to the high voltage-side is

$$Z_{e1} = 0.2 + j0.45 + \left(\frac{2400}{240}\right)^2 (0.002 + j0.0045) = 0.4 + j0.9 \ \ \Omega$$

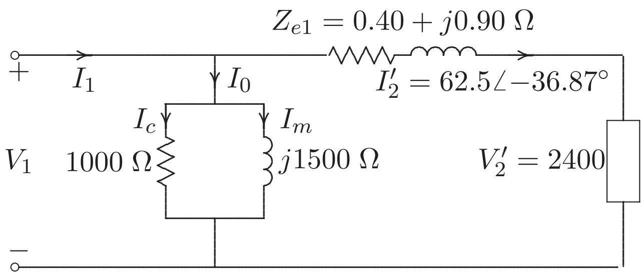


FIGURE 20
Equivalent circuit referred to the high-voltage side.

(b) At full-load 0.8 power factor lagging $S = 150\angle 36.87^\circ$ kVA, and the referred primary current is

$$I_2{}' = \frac{S^*}{V^*} = \frac{150000\angle{-36.87^\circ}}{2400} = 62.5\angle{-36.87^\circ} \ \ \text{A}$$

$$V_1 = 2400 + (0.4 + j0.9)(62.5\angle{-36.87^\circ}) = 2453.933\angle 0.7^\circ \ \ \text{V}$$

The voltage regulation is

$$V.R. = \frac{2453.933 - 2400}{2400} \times 100 = 2.247\%$$

(c) At full-load 0.8 power factor leading $S = 150\angle{-36.87^\circ}$ kVA, and the referred primary current is

$$I_2{}' = \frac{S^*}{V^*} = \frac{150000\angle{+36.87^\circ}}{2400} = 62.5\angle{+36.87^\circ} \ \ \text{A}$$

$$V_1 = 2400 + (0.4 + j0.9)(62.5\angle{+36.87^\circ}) = 2387.004\angle 1.44^\circ \ \ \text{V}$$

The voltage regulation is

$$V.R. = \frac{2453.933 - 2400}{2400} \times 100 = -0.541\%$$

(d) Entering **trans** at the *MATLAB* prompt, result in

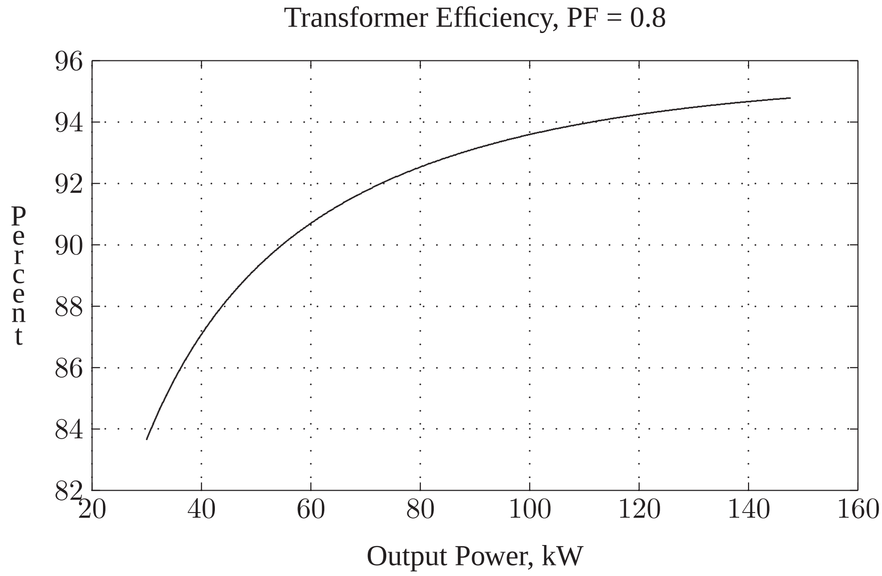


FIGURE 21
Efficiency curve of Problem 3.5.

```
                  TRANSFORMER ANALYSIS

Type of parameters for input                Select
----------------------------                ------
To obtain equivalent circuit from tests        1
To input individual winding impedances         2
To input transformer equivalent impedance      3
To quit                                        0
Select number of menu --> 2

Enter Transformer rated power in kVA, S = 150
Enter rated low voltage in volts  = 240
Enter rated high voltage in volts = 2400
```

```
Enter LV winding series impedance R1+j*X1 in ohm=0.002+j*.0045
Enter HV winding series impedance R2+j*X2 in ohm=0.2+j*0.45
Enter 'lv' within quotes for low side shunt branch or
enter 'hv' within quotes for high side shunt branch -> 'hv'
Shunt resistance in ohm (if neglected enter inf) = 1000
Shunt reactance in ohm (if neglected enter inf) = 1500

Shunt branch ref. to LV side      Shunt branch ref. to HV side
    Rc =       10.000 ohm             Rc =     1000.000 ohm
    Xm =       15.000 ohm             Xm =     1500.000 ohm

Series branch ref. to LV side     Series branch ref.to HV side
    Ze =  0.0040 + j  0.0090 ohm    Ze =  0.4000 + j  0.90 ohm

Hit return to continue
    Enter load kVA, S2 = 150
    Enter load power factor, pf = 0.8
    Enter 'lg' within quotes for lagging pf
    or 'ld' within quotes for leading pf -> 'lg'
    Enter load terminal voltage in volt, V2 = 240
    Secondary load voltage  =    240.000 V
    Secondary load current  =    625.000 A  at -36.87 degrees
    Current ref. to primary =     62.500 A  at -36.87 degrees
    Primary no-load current =      2.949 A  at -33.69 degrees
    Primary input current   =     65.445 A  at -36.73 degrees
    Primary input voltage   =   2453.933 V  at  0.70  degrees
    Voltage regulation      =      2.247 percent
    Transformer efficiency  =     94.249 percent
    Maximum efficiency is 95.238 %occurs at 288 kVA with 0.8pf
```

The efficiency curve is shown in Figure 21. The analysis is repeated for the full-load kVA, 0.8 power factor leading.

3.6. A 60-kVA, 4800/2400-V single-phase transformer gave the following test results:

1. Rated voltage is applied to the low voltage winding and the high voltage winding is open-circuited. Under this condition, the current into the low voltage winding is 2.4 A and the power taken from the 2400 V source is 3456 W.

2. A reduced voltage of 1250 V is applied to the high voltage winding and the low voltage winding is short-circuited. Under this condition, the current flowing into the high voltage winding is 12.5 A and the power taken from the 1250 V source is 4375 W.

(a) Determine parameters of the equivalent circuit referred to the high voltage side.

(b) Determine voltage regulation and efficiency when transformer is operating at full-load, 0.8 power factor lagging, and a terminal voltage of 2400 V.

(c) What is the load kVA for maximum efficiency and the maximum efficiency at 0.8 power factor?

(d) Determine the efficiency when transformer is operating at 3/4 full-load, 0.8 power factor lagging, and a terminal voltage of 2400 V.

(e) Verify your answers by running the **trans** program in *MATLAB* and obtain the transformer efficiency curve.

(a) From the no-load test data

$$R_{cLV} = \frac{|V_2|^2}{P_0} = \frac{(2400)^2}{3456} = 1666.667\ \Omega$$

$$I_c = \frac{|V_2|}{R_{cLV}} = \frac{2400}{1666.67} = 1.44\ \ \text{A}$$

$$I_m = \sqrt{I_0^2 - I_c^2} = \sqrt{(2.4)^2 - (1.44)^2} = 1.92\ \ \text{A}$$

$$X_{mLV} = \frac{|V_2|}{I_m} = \frac{2400}{1.92} = 1250\ \ \Omega$$

The shunt parameters referred to the high-voltage side are

$$R_{CHV} = \left(\frac{4800}{2400}\right)^2 1666.667 = 6666.667\ \Omega$$

$$X_{mHV} = \left(\frac{4800}{2400}\right)^2 1250 = 5000\ \Omega$$

From the short-circuit test data

$$Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{1250}{12.5} = 100\ \Omega$$

$$R_{e1} = \frac{P_{sc}}{(I_{sc})^2} = \frac{4375}{(12.5)^2} = 28\ \Omega$$

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{(100)^2 - (28)^2} = 96 \ \ \Omega$$

(b) At full-load 0.8 power factor lagging $S = 60\angle 36.87^\circ$ kVA, and the referred primary current is

$$I_2{}' = \frac{S^*}{V^*} = \frac{60000\angle{-36.87^\circ}}{4800} = 12.5\angle{-36.87^\circ} \ \ \text{A}$$

$$V_1 = 4800 + (28 + j96)(12.5\angle{-36.87^\circ}) = 5848.29\angle 7.368^\circ \ \ \text{V}$$

The voltage regulation is

$$V.R. = \frac{5848.29 - 4800}{4800} \times 100 = 21.839\%$$

Efficiency at full-load, 0.8 power factor is

$$\eta = \frac{(60)(0.8)}{(60)(0.8) + 3.456 + 4.375} \times 100 = 85.974\ \%$$

(c) At Maximum efficiency $P_c = P_{cu}$, and we have

$$\begin{aligned} P_{cu} &\propto S_{fl}^2 \\ P_c &\propto S_{max\eta}^2 \end{aligned}$$

Therefore, the load kVA at maximum efficiency is

$$S_{max\eta} = \sqrt{\frac{P_c}{P_{cu}}} S_{fl} = \sqrt{\frac{3456}{4375}}\ 60 = 53.327 \ \ \text{kVA}$$

and the maximum efficiency at 0.8 power factor is

$$\eta = \frac{(53.327)(0.8)}{(53.327)(0.8) + 3.456 + 3.456} \times 100 = 86.057\ \%$$

(d) At $3/4$ full-load, the copper loss is

$$P_{cu3/4} = (3/4)^2(4375) = 2460.9375 \ \ \text{W}$$

Therefore, the efficiency at $3/4$ full-load, 0.8 power factor is

$$\eta = \frac{(45)(0.8)}{(45)(0.8) + 3.456 + 2.4609} \times 100 = 85.884\ \%$$

(e) The commands **trans** display the following menu

```
                    TRANSFORMER ANALYSIS

Type of parameters for input                Select
----------------------------                ------
To obtain equivalent circuit from tests        1
To input individual winding impedances         2
To input transformer equivalent impedance      3
To quit                                        0
Select number of menu --> 1

Enter Transformer rated power in kVA, S = 60
Enter rated low voltage in volts  = 2400
Enter rated high voltage in volts = 4800

Open circuit test data
----------------------
Enter 'lv' within quotes for data referred to low side or
enter 'hv' within quotes for data referred to high side ->'lv'
Enter input voltage in volts, Vo = 2400
Enter no-load current in Amp, Io = 2.4
Enter no-load input power in Watt, Po = 3456

Short circuit test data
------------------------
Enter 'lv' within quotes for data referred to low side or
enter 'hv' within quotes for data referred to high side ->'hv'
Enter reduced input voltage in volts, Vsc = 1250
Enter input current in Amp, Isc = 12.5
Enter input power in Watt, Psc = 4375

Shunt branch ref. to LV side      Shunt branch ref. to HV side
    Rc =     1666.667 ohm             Rc =     6666.667 ohm
    Xm =     1250.000 ohm             Xm =     5000.000 ohm

Series branch ref. to LV side     Series branch ref.to HV side
    Ze = 7.0000 + j  24.0000 ohm    Ze = 28.000 + j 96.000 ohm
Hit return to continue
    Enter load KVA, S2 = 60
    Enter load power factor, pf = 0.8
    Enter 'lg' within quotes for lagging pf
    or 'ld' within quotes for leading pf -> 'lg'
    Enter load terminal voltage in volt, V2 = 2400
    Secondary load voltage  =   2400.000 V
    Secondary load current  =     25.000 A  at -36.87 degrees
```

```
Current ref. to primary =     12.500 A  at -36.87 degrees
Primary no-load current =      1.462 A  at -53.13 degrees
Primary input current   =     13.910 A  at -38.56 degrees
Primary input voltage   =   5848.290 V  at  7.37  degrees
Voltage regulation      =     21.839 percent
Transformer efficiency  =     85.974 percent
Maximum efficiency is 86.057%occurs at 53.33kVA with 0.8pf
```

The efficiency curve is shown in Figure 22. The analysis is repeated for the $3/4$ full-load kVA, 0.8 power factor lagging.

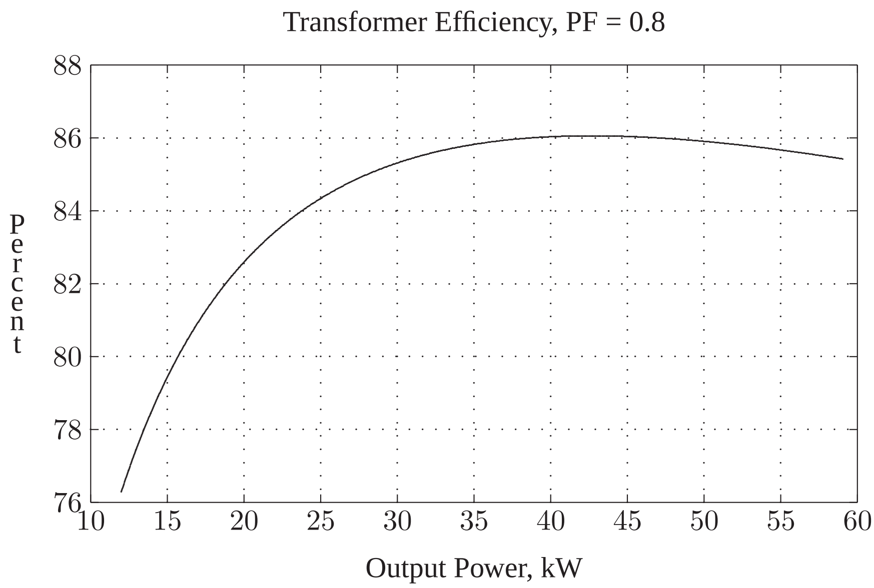


FIGURE 22
Efficiency curve of Problem 3.6.

3.7. A two-winding transformer rated at 9-kVA, 120/90-V, 60-HZ has a core loss of 200 W and a full-load copper loss of 500 W.

(a) The above transformer is to be connected as an auto transformer to supply a load at 120 V from 210 V source. What kVA load can be supplied without exceeding the current rating of the windings? (For this part assume an ideal transformer.)

(b) Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

The two-winding transformer rated currents are:

$$I_{LV} = \frac{9000}{90} = 100 \ \text{A}$$

$$I_{HV} = \frac{9000}{120} = 75 \text{ A}$$

The auto transformer connection is as shown in Figure 23.

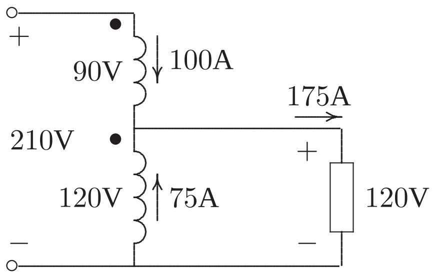


FIGURE 23
Auto transformer connection for Problem 3.7.

(a) With windings carrying rated currents, the auto transformer rating is

$$S = (120)(175)(10^{-3}) = 21 \text{ kVA}$$

(b) Since the windings are subjected to the same rated voltages and currents as the two-winding transformer, the auto transformer copper loss and the core loss at the rated values are the same as the two-winding transformer. Therefore, the auto transformer efficiency at 0.8 power factor is

$$\eta = \frac{(21000)(0.8)}{(21000)(0.8) + 200 + 500} \times 100 = 96 \text{ \%}$$

3.8. Three identical 9-MVA, 7.2-kV/4.16-kV, single-phase transformers are connected in Y on the high-voltage side and Δ on the low voltage side. The equivalent series impedance of each transformer referred to the high-voltage side is $0.12 + j0.82\ \Omega$ per phase. The transformer supplies a balanced three-phase load of 18 MVA, 0.8 power factor lagging at 4.16 kV. Determine the line-to-line voltage at the high-voltage terminals of the transformer.

The per phase equivalent circuit is shown in Figure 24. The load complex power is

$$S = 18000\angle 36.87^\circ \text{ kVA}$$

$$I_1 = \frac{18000\angle{-36.87^\circ}}{(3)(7.2)} = 833.333\angle{-36.87^\circ} \text{ A}$$

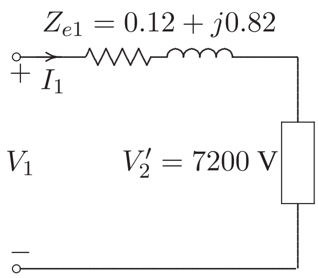


FIGURE 24
The per phase equivalent circuit of Problem 3.8.

$$V_1 = 7.2\angle 0^\circ + (0.12 + j0.82)(833.333\angle{-36.87^\circ})(10^{-3}) = 7.7054\angle 3.62^\circ \text{ kV}$$

Therefore, the magnitude of the primary line-to-line supply voltage is $V_{1LL} = \sqrt{3}(7.7054) = 13.346$ kV.

3.9. A 400-MVA, 240-kV/24-KV, three-phase Y-Δ transformer has an equivalent series impedance of $1.2 + j6\ \Omega$ per phase referred to the high-voltage side. The transformer is supplying a three-phase load of 400-MVA, 0.8 power factor lagging at a terminal voltage of 24 kV (line to line) on its low-voltage side. The primary is supplied from a feeder with an impedance of $0.6 + j1.2\ \Omega$ per phase. Determine the line-to-line voltage at the high-voltage terminals of the transformer and the sending-end of the feeder.

The per phase equivalent circuit referred to the primary is is shown in Figure 25. The load complex power is

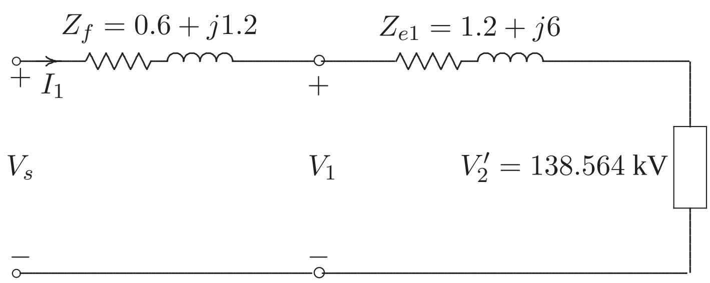


FIGURE 25
The per phase equivalent circuit for Problem 3.9.

$$S = 400000\angle 36.87^\circ \text{ kVA}$$

and the referred secondary voltage per phase is

$$V_2' = \frac{240}{\sqrt{3}} = 138.564 \ \text{kV}$$

$$I_1 = \frac{400000\angle{-36.87^\circ}}{(3)(138.564)} = 962.25\angle{-36.87^\circ} \ \text{A}$$

$$V_1 = 138.564\angle 0^\circ + (1.2 + j6)(962.25\angle{-36.87^\circ})(10^{-3}) = 143\angle 1.57^\circ \ \text{kV}$$

Therefore, the line-to-line voltage magnitude at the high voltage terminal of the transformer is $V_{1LL} = \sqrt{3}(143) = 247.69$ kV.

$$V_s = 138.564\angle 0^\circ + (1.8 + j7.2)(962.25\angle{-36.87^\circ})(10^{-3}) = 144.177\angle 1.79^\circ \text{ kV}$$

Therefore, the line-to-line voltage magnitude at the sending end of the feeder is $V_{1LL} = \sqrt{3}(143) = 249.72$ kV.

3.10. In Problem 3.9, with transformer rated values as base quantities, express all impedances in per unit. Working with per-unit values, determine the line-to-line voltage at the high-voltage terminals of the transformer and the sending-end of the feeder.

With the transformer rated MVA as base, the HV-side base impedance is

$$Z_B = \frac{(240)^2}{400} = 144 \ \Omega$$

The transformer and the feeder impedances in per unit are

$$\begin{aligned} Z_e &= \frac{(1.2 + j6)}{144} = 0.008333 + j0.04166 \text{ pu} \\ Z_f &= \frac{(0.6 + j1.2)}{144} = 0.004166 + j0.00833 \text{ pu} \end{aligned}$$

$$I = \frac{1\angle{-36.87^\circ}}{1\angle 0^\circ} = 1\angle{-36.87^\circ} \text{ pu}$$

$$V_1 = 1.0\angle 0^\circ + (0.008333 + j0.04166)(1\angle{-36.87^\circ} = 1.03205\angle 1.57^\circ \text{ pu}$$

Therefore, the line-to-line voltage magnitude at the high-voltage terminal of the transformer is $(1.03205)(240) = 247.69$ kV

$$V_s = 1.0\angle 0^\circ + (0.0125 + j0.05)(1\angle{-36.87^\circ} = 1.0405\angle 1.79^\circ \text{ pu}$$

Therefore, the line-to-line voltage magnitude at the high voltage terminal of the transformer is $(1.0405)(240) = 249.72$ kV

3.11. A three-phase, Y-connected, 75-MVA, 27-kV synchronous generator has a synchronous reactance of $9.0\ \Omega$ per phase. Using rated MVA and voltage as base values, determine the per unit reactance. Then refer this per unit value to a 100-MVA, 30-kV base.

The base impedance is

$$Z_B = \frac{(KV_B)^2}{MVA_B} = \frac{(27)^2}{75} = 9.72\ \Omega$$

$$X_{pu} = \frac{9}{9.72} = 0.926 \ \text{ pu}$$

The generator reactance on a 100-MVA, 30-kV base is

$$X_{pu_{new}} = 0.926\left(\frac{100}{75}\right)\left(\frac{27}{30}\right)^2 = 1.0 \ \text{ pu}$$

3.12. A 40-MVA, 20-kV/400-kV, single-phase transformer has the following series impedances:

$Z_1 = 0.9 + j1.8\ \Omega$ and $Z_2 = 128 + j288\ \Omega$

Using the transformer rating as base, determine the per unit impedance of the transformer from the ohmic value referred to the low-voltage side. Compute the per unit impedance using the ohmic value referred to the high-voltage side.

The transformer equivalent impedance referred to the low-voltage side is

$$Z_{e1} = 0.9 + j1.8 + \left(\frac{20}{400}\right)^2(128 + j288) = 1.22 + j2.52\ \Omega$$

The low-voltage base impedance is

$$Z_{B1} = \frac{(20)^2}{40} = 10\ \Omega$$

$$Z_{pu_1} = \frac{1.22 + j2.52}{10} = 0.122 + j0.252 \ \text{ pu}$$

The transformer equivalent impedance referred to the high-voltage side is

$$Z_{e2} = \left(\frac{400}{20}\right)^2 (0.9 + j1.8) + (128 + j288) = 488 + j1008\ \Omega$$

The high-voltage base impedance is

$$Z_{B2} = \frac{(400)^2}{40} = 4000\ \Omega$$

$$Z_{pu_2} = \frac{488 + j1008}{4000} = 0.122 + j0.252\ \text{ pu}$$

We note that the transformer per unit impedance has the same value regardless of whether it is referred to the primary or the secondary side.

3.13. Draw an impedance diagram for the electric power system shown in Figure 26 showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

G_1 :	90 MVA	20 kV	$X = 9\%$
T_1 :	80 MVA	20/200 kV	$X = 16\%$
T_2 :	80 MVA	200/20 kV	$X = 20\%$
G_2 :	90 MVA	18 kV	$X = 9\%$
Line:		200 kV	$X = 120\ \Omega$
Load:		200 kV	$S = 48$ MW $+j64$ Mvar

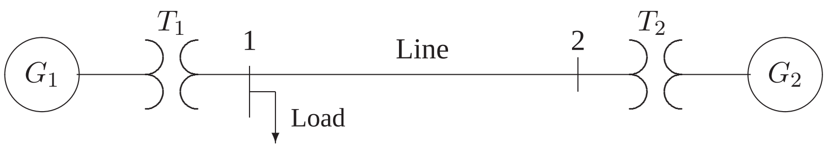


FIGURE 26
One-line diagram for Problem 3.13

The base voltage V_{BG1}on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{BG2} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$G: \qquad X = 0.09\left(\frac{100}{90}\right) = 0.10 \text{ pu}$$
$$T_1: \qquad X = 0.16\left(\frac{100}{80}\right) = 0.20 \text{ pu}$$
$$T_2: \qquad X = 0.20\left(\frac{100}{80}\right) = 0.25 \text{ pu}$$
$$G_2: \qquad X = 0.09\left(\frac{100}{90}\right)\left(\frac{18}{20}\right)^2 = 0.081 \text{ pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line reactance is

$$Line: \qquad X = \left(\frac{120}{400}\right) = 0.30 \text{ pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(200)^2}{48 - j64} = 300 + j400 \ \Omega$$

The load impedance in per unit is

$$Z_{L(pu)} = \frac{300 + j400}{400} = 0.75 + j1.0 \text{ pu}$$

The per unit equivalent circuit is shown in Figure 27.

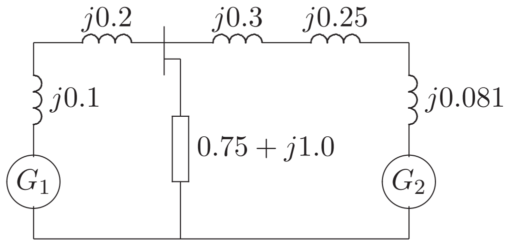


FIGURE 27
Per unit impedance diagram for Problem 3.11.

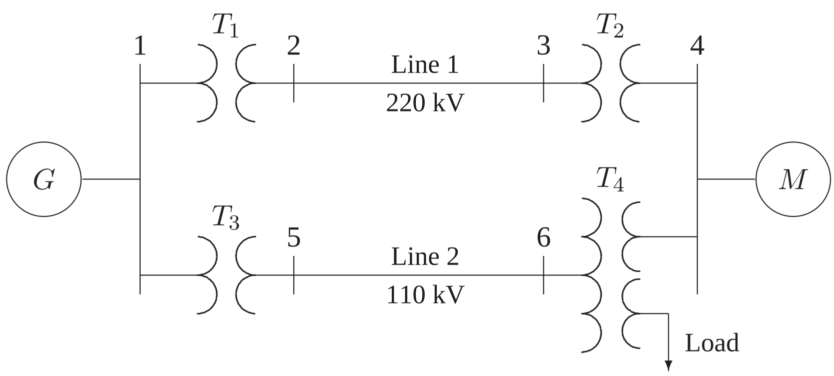


FIGURE 28
One-line diagram for Problem 3.14

3.14. The one-line diagram of a power system is shown in Figure 28.
The three-phase power and line-line ratings are given below.

G:	80 MVA	22 kV	$X = 24\%$
T_1:	50 MVA	22/220 kV	$X = 10\%$
T_2:	40 MVA	220/22 kV	$X = 6.0\%$
T_3:	40 MVA	22/110 kV	$X = 6.4\%$
Line 1:		220 kV	$X = 121\ \Omega$
Line 2:		110 kV	$X = 42.35\ \Omega$
M:	68.85 MVA	20 kV	$X = 22.5\%$
Load:	10 Mvar	4 kV	Δ-connected capacitors

The three-phase ratings of the three-phase transformer are

Primary:	Y-connected	40MVA, 110 kV
Secondary:	Y-connected	40 MVA, 22 kV
Tertiary:	Δ-connected	15 MVA, 4 kV

The per phase measured reactances at the terminal of a winding with the second one short-circuited and the third open-circuited are

$Z_{PS} = 9.6\%$	40 MVA, 110 kV / 22 kV
$Z_{PT} = 7.2\%$	40 MVA, 110 kV / 4 kV
$Z_{ST} = 12\%$	40 MVA, 22 kV / 4 kV

Obtain the T-circuit equivalent impedances of the three-winding transformer to the common MVA base. Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 22 kV as the voltage base for generator.

The base voltage V_{B1} on the LV side of T_1 is 22 kV. Hence the base on its HV side

is

$$V_{B2} = 22\left(\frac{220}{22}\right) = 220 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B3} = 220$ kV, and on its LV side at

$$V_{B4} = 220\left(\frac{22}{220}\right) = 22 \quad \text{kV}$$

Similarly, the voltage base at buses 5 and 6 are

$$V_{B5} = V_{B6} = 22\left(\frac{110}{22}\right) = 110 \quad \text{kV}$$

Voltage base for the tertiary side of T_4 is

$$V_{BT} = 110\left(\frac{4}{110}\right) = 4 \quad \text{kV}$$

The per unit impedances on a 100 MVA base are:

$$\begin{aligned}
G&: \qquad X = 0.24\left(\frac{100}{80}\right) = 0.30 \ \text{pu} \\
T_1&: \qquad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{pu} \\
T_2&: \qquad X = 0.06\left(\frac{100}{40}\right) = 0.15 \ \text{pu} \\
T_3&: \qquad X = 0.064\left(\frac{100}{40}\right) = 0.16 \quad \text{pu}
\end{aligned}$$

The motor reactance is expressed on its nameplate rating of 68.85 MVA, and 20 kV. However, the base voltage at bus 4 for the motor is 22 kV, therefore

$$M: \qquad X = 0.225\left(\frac{100}{68.85}\right)\left(\frac{20}{22}\right)^2 = 0.27 \ \text{pu}$$

Impedance bases for lines 1 and 2 are

$$\begin{aligned}
Z_{B2} &= \frac{(220)^2}{100} = 484 \ \Omega \\
Z_{B5} &= \frac{(110)^2}{100} = 121 \ \Omega
\end{aligned}$$

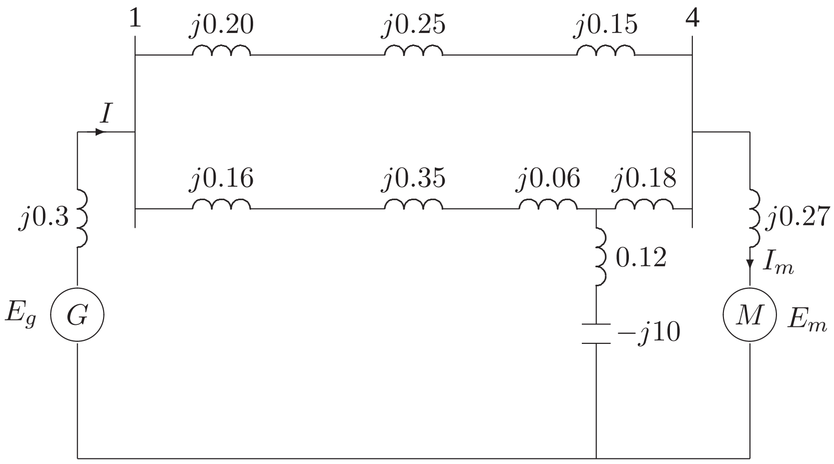


FIGURE 29
Per unit impedance diagram for Problem 3.14.

Line 1 and 2 per unit reactances are

$$Line_1: \qquad X = \left(\frac{121}{484}\right) = 0.25 \ \text{pu}$$
$$Line_2: \qquad X = \left(\frac{42.35}{121}\right) = 0.35 \ \text{pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(4)^2}{j10} = -j1.6 \ \ \Omega$$

The base impedance for the load is

$$Z_{BT} = \frac{(4)^2}{100} = 0.16 \ \ \Omega$$

Therefore, the load impedance in per unit is

$$Z_{L(pu)} = \frac{-j1.6}{0.16} = -j10 \ \text{pu}$$

The three-winding impedances on a 100 MVA base are

$$Z_{PS} = 0.096\left(\frac{100}{40}\right) = 0.24 \ \text{pu}$$
$$Z_{PT} = 0.072\left(\frac{100}{40}\right) = 0.18 \ \text{pu}$$
$$Z_{ST} = 0.120\left(\frac{100}{40}\right) = 0.30 \ \text{pu}$$

The equivalent T circuit impedances are

$$Z_P = \frac{1}{2}(j0.24 + j0.18 - j0.30) = j0.06 \text{ pu}$$
$$Z_S = \frac{1}{2}(j0.24 + j0.30 - j0.18) = j0.18 \text{ pu}$$
$$Z_T = \frac{1}{2}(j0.18 + j0.30 - j0.24) = j0.12 \text{ pu}$$

The per unit equivalent circuit is shown in Figure 29.

3.15. The three-phase power and line-line ratings of the electric power system shown in Figure 30 are given below.

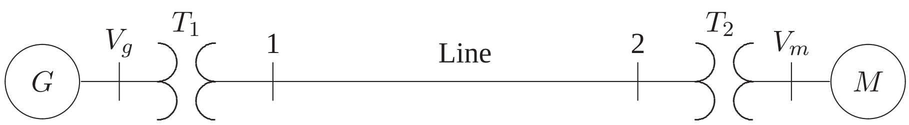


FIGURE 30
One-line diagram for Problem 3.15

G_1 :	60 MVA	20 kV	$X = 9\%$
T_1 :	50 MVA	20/200 kV	$X = 10\%$
T_2 :	50 MVA	200/20 kV	$X = 10\%$
M :	43.2 MVA	18 kV	$X = 8\%$
Line:		200 kV	$Z = 120 + j200\ \Omega$

(a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

(b) The motor is drawing 45 MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

The base voltage V_{BG1}on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{Bm} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$
\begin{aligned}
G&: \qquad X = 0.09\left(\frac{100}{60}\right) = 0.15 \ \text{pu}\\
T_1&: \qquad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{pu}\\
T_2&: \qquad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{pu}\\
M&: \qquad X = 0.08\left(\frac{100}{43.2}\right)\left(\frac{18}{20}\right)^2 = 0.15 \ \text{pu}
\end{aligned}
$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line impedance is

$$Line: \qquad Z_{line} = (\frac{120 + j200}{400}) = 0.30 + j0.5 \ \text{pu}$$

The per unit equivalent circuit is shown in Figure 31.

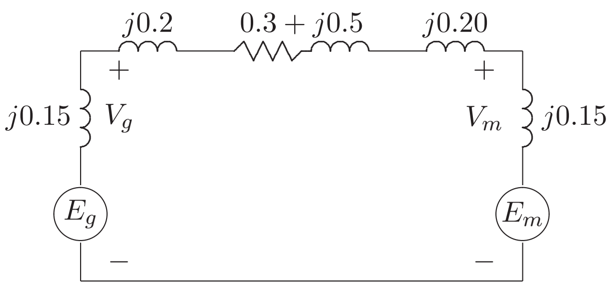


FIGURE 31
Per unit impedance diagram for Problem 3.15.

(b) The motor complex power in per unit is

$$S_m = \frac{45\angle 36.87^\circ}{100} = 0.45\angle 36.87^\circ \ \text{pu}$$

and the motor terminal voltage is

$$V_m = \frac{18\angle 0^\circ}{20} = 0.90\angle 0^\circ \ \text{pu}$$

$$I = \frac{0.45\angle{-36.87^\circ}}{0.90\angle 0^\circ} = 0.5\angle{-36.87^\circ} \ \text{ pu}$$

$$V_g = 0.90\angle 0^\circ + (0.3 + j0.9)(0.5\angle{-36.87^\circ} = 1.31795\angle 11.82^\circ \ \text{ pu}$$

Thus, the generator line-to-line terminal voltage is

$$V_g = (1.31795)(20) = 26.359 \ \text{ kV}$$

$$E_g = 0.90\angle 0^\circ + (0.3 + j1.05)(0.5\angle{-36.87^\circ} = 1.375\angle 13.88^\circ \ \text{ pu}$$

Thus, the generator line-to-line internal emf is

$$E_g = (1.375)(20) = 27.5 \ \text{ kV}$$

3.16. The one-line diagram of a three-phase power system is as shown in Figure 32. Impedances are marked in per unit on a 100-MVA, 400-kV base. The load at bus 2 is $S_2 = 15.93$ MW $-j33.4$ Mvar, and at bus 3 is $S_3 = 77$ MW $+j14$ Mvar. It is required to hold the voltage at bus 3 at $400\angle 0^\circ$ kV. Working in per unit, determine the voltage at buses 2 and 1.

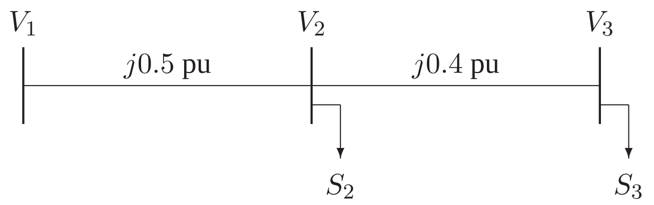


FIGURE 32
One-line diagram for Problem 3.16

$$\begin{aligned}
S_2 &= 15.93 \text{ MW} - j33.4 \text{ Mvar} = 0.1593 - j0.334 \ \text{ pu} \\
S_3 &= 77.00 \text{ MW} + j14.0 \text{ Mvar} = 0.7700 + j0.140 \ \text{ pu} \\
V_3 &= \frac{400\angle 0^\circ}{400} = 1.0\angle 0^\circ \ \text{ pu} \\
I_3 &= \frac{S_3^*}{V_3^*} = \frac{0.77 - j0.14}{1.0\angle 0^\circ} = 0.77 - j0.14 \ \text{ pu} \\
V_2 &= 1.0\angle 0^\circ + (j0.4)(0.77 - j0.14) = 1.1\angle 16.26^\circ \ \text{ pu}
\end{aligned}$$

Therefore, the line-to-line voltage at bus 2 is

$$
\begin{aligned}
V_2 &= (400)(1.1) = 440 \;\text{ kV} \\
I_2 &= \frac{S_2^*}{V_2^*} = \frac{0.1593 + j0.334}{1.1\angle{-16.26^\circ}} = 0.054 + j0.332 \;\text{ pu} \\
I_{12} &= (0.77 - j0.14) + (0.054 + j0.332) = 0.824 + j0.192 \;\text{ pu} \\
V_1 &= 1.1\angle 16.26^\circ + (j0.5)(0.824 + j0.192) = 1.2\angle 36.87^\circ \;\text{ pu}
\end{aligned}
$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (400)(1.2) = 480 \;\text{ kV}$$

3.17. The one-line diagram of a three-phase power system is as shown in Figure 33. The transformer reactance is 20 percent on a base of 100-MVA, 23/115-kV and the line impedance is $Z = j66.125\Omega$. The load at bus 2 is $S_2 = 184.8$ MW $+j6.6$ Mar, and at bus 3 is $S_3 = 0$ MW $+j20$ Mar. It is required to hold the voltage at bus 3 at $115\angle 0^\circ$ kV. Working in per unit, determine the voltage at buses 2 and 1.

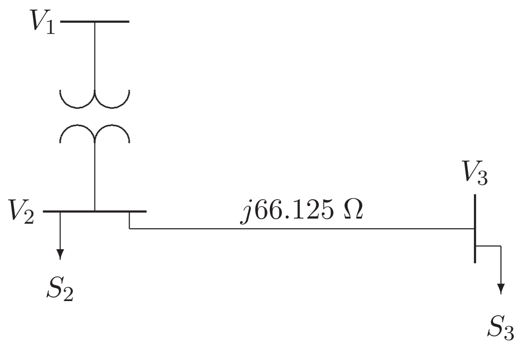


FIGURE 33
One-line diagram for Problem 3.17

$$
\begin{aligned}
S_2 &= 184.8 \text{ MW} + j6.6 \text{ Mvar} = 1.848 + j0.066 \;\text{ pu} \\
S_3 &= \quad 0 \text{ MW} + j20.0 \text{ Mvar} = \quad 0 + j0.20 \;\text{ pu}
\end{aligned}
$$

$$
\begin{aligned}
V_3 &= \frac{115\angle 0^\circ}{115} = 1.0\angle 0^\circ \;\text{ pu} \\
I_3 &= \frac{S_3^*}{V_3^*} = \frac{-j0.2}{1.0\angle 0^\circ} = -j0.2 \;\text{ pu} \\
V_2 &= 1.0\angle 0^\circ + (j0.5)(-j0.2) = 1.1\angle 0^\circ \;\text{ pu}
\end{aligned}
$$

Therefore, the line-to-line voltage at bus 2 is

$$V_2 = (115)(1.1) = 126.5 \text{ kV}$$

$$I_2 = \frac{S_2^*}{V_2^*} = \frac{1.848 - j0.066}{1.1\angle 0^\circ} = 1.68 - j0.06 \text{ pu}$$
$$I_{12} = (1.68 - j0.06) + (-j0.2) = 1.68 - j0.26 \text{ pu}$$
$$V_1 = 1.1\angle 0^\circ + (j0.2)(1.68 - j0.26) = 1.2\angle 16.26^\circ \text{ pu}$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (23)(1.2) = 27.6 \text{ kV}$$

CHAPTER 4 PROBLEMS

4.1. A solid cylindrical aluminum conductor 25 km long has an area of 336,400 circular mils. Obtain the conductor resistance at (a) $20°$C and (b) $50°$C. The resistivity of aluminum at $20°$C is 2.8×10^{-8} Ω-m.

(a)

$$d = \sqrt{336400} = 580 \text{ mil} = (580)(10^{-3})(2.54) = 1.4732 \text{ cm}$$

$$A = \frac{\pi d^2}{4} = 1.704564 \text{ cm}^2$$

$$R_1 = \rho\,\frac{\ell}{A} = 2.8 \times 10^{-8}\,\frac{25 \times 10^3}{1.704564 \times 10^{-4}} = 4.106\ \Omega$$

(b)

$$R_2 = R_1\left(\frac{228 + 50}{228 + 20}\right) = 4.106\left(\frac{278}{248}\right) = 4.6\ \Omega$$

4.2. A transmission-line cable consists of 12 identical strands of aluminum, each 3 mm in diameter. The resistivity of aluminum strand at $20°$C is 2.8×10^{-8} Ω-m. Find the $50°$C AC resistance per Km of the cable. Assume a skin-effect correction factor of 1.02 at 60 Hz.

$$A = 12\,\frac{\pi d^2}{4} = 12\,\frac{(\pi)(3^2)}{4} = 84.823 \;\; \text{mm}^2$$

$$R_{20} = \rho\,\frac{\ell}{A} = \frac{2.8 \times 10^{-8} \times 10^3}{84.823 \times 10^{-6}} = 0.33\;\Omega$$

$$R_{50} = R_{20}\left(\frac{228+50}{228+20}\right) = 0.33\left(\frac{278}{248}\right) = 0.37\;\Omega$$

$$R_{AC} = (1.02)(0.37) = 0.3774\;\Omega$$

4.3. A three-phase transmission line is designed to deliver 190.5-MVA at 220-kV over a distance of 63 Km. The total transmission line loss is not to exceed 2.5 percent of the rated line MVA. If the resistivity of the conductor material is 2.84×10^{-8} Ω-m, determine the required conductor diameter and the conductor size in circular mils.

The total transmission line loss is

$$P_L = \frac{2.5}{100}(190.5) = 4.7625 \;\; \text{MW}$$

$$|I| = \frac{S}{\sqrt{3}\,V_L} = \frac{(190.5)10^3}{\sqrt{3}(220)} = 500 \;\; \text{A}$$

From $P_L = 3R|I|^2$, the line resistance per phase is

$$R = \frac{4.7625 \times 10^6}{3(500)^2} = 6.35\;\Omega$$

The conductor cross sectional area is

$$A = \frac{(2.84 \times 10^{-8})(63 \times 10^3)}{6.35} = 2.81764 \times 10^{-4} \;\; \text{m}^2$$

Therefore

$$d = 1.894 \text{ cm} = 0.7456 \text{ in} = 556000 \;\; \text{cmil}$$

4.4. A single-phase transmission line 35 Km long consists of two solid round conductors, each having a diameter of 0.9 cm. The conductor spacing is 2.5 m. Calculate the equivalent diameter of a fictitious hollow, thin-walled conductor having the same equivalent inductance as the original line. What is the value of the inductance per conductor?

$$r^{'} = e^{-\frac{1}{4}}r = e^{-\frac{1}{4}}\left(\frac{0.9}{2}\right) = 0.35 \text{ cm}$$

or $\quad d = 0.7$ cm.

$$L = 0.2 \ln \frac{2.5}{0.35 \times 10^{-2}} = 1.314 \text{ mH/Km}$$

The inductance per conductor is $\ell L = (35)(1.314) = 46$ mH.

4.5. Find the geometric mean radius of a conductor in terms of the radius r of an individual strand for
(a) Three equal strands as shown in Figure 34(a)
(b) Four equal strands as shown in Figure 34(b)

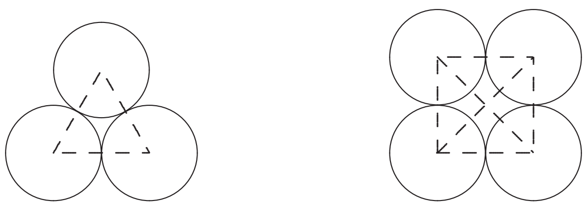

(a) (b)

FIGURE 34
Cross section of the stranded conductor for Problem 4.5.

(a)

$$\begin{aligned} GMR &= \sqrt[9]{(r'2r \times 2r)^3} \\ &= \sqrt[3]{e^{-\frac{1}{4}}r \times 2r \times 2r} = 1.46r \end{aligned}$$

(b)

$$\begin{aligned} GMR &= \sqrt[16]{(r' \times 2r \times 2r \times \sqrt{2}\, 2r)^4} \\ &= \sqrt[4]{e^{-\frac{1}{4}} 8\sqrt{2}\, r^4} = 1.723r \end{aligned}$$

4.6. One circuit of a single-phase transmission line is composed of three solid 0.5-cm radius wires. The return circuit is composed of two solid 2.5-cm radius wires. The arrangement of conductors is as shown in Figure 35. Applying the concept of the GMD and GMR, find the inductance of the complete line in millihenry per kilometer.

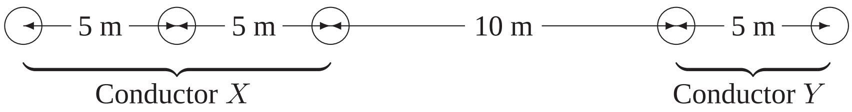


FIGURE 35
Conductor layout for Problem 4.6.

$$D_m = \sqrt[6]{(20)(25)(15)(20)(10)(15)} = 16.802 \ \text{m}$$

$$D_{SX} = \sqrt[9]{(e^{-\frac{1}{4}} \times 0.005 \times 5 \times 10)(e^{-\frac{1}{4}} \times 0.005 \times 5 \times 5)(e^{-\frac{1}{4}} \times 0.005 \times 5 \times 10)} = 0.5366 \ \text{m}$$

$$D_{SY} = \sqrt[4]{\left(e^{-\frac{1}{4}} \times 0.025 \times 5\right)^2} = 0.312 \ \text{m}$$

Therefore

$$L_X = 0.2 \ln \frac{16.802}{0.5366} = 0.6888 \ \text{mH/Km}$$

and

$$L_Y = 0.2 \ln \frac{16.802}{0.312} = 0.79725 \ \text{mH/Km}$$

The loop inductance is

$$L = L_X + L_Y = 0.6888 + 0.79725 = 1.48605 \ \text{mH/Km}$$

4.7. A three-phase, 60-Hz transposed transmission line has a flat horizontal configuration as shown in Figure 36. The line reactance is 0.486 Ω per kilometer. The conductor geometric mean radius is 2.0 cm. Determine the phase spacing D in meter.

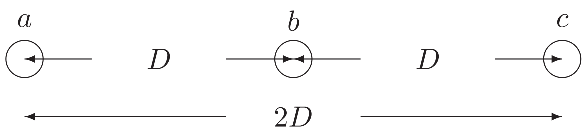


FIGURE 36
Conductor layout for Problem 4.7.

$$L = \frac{(0.486)10^3}{(2\pi)(60)} = 1.2892 \text{ mH/Km}$$

Therefore, we have

$$1.2892 = 0.2 \ln \frac{GMD}{0.02} \quad \text{or} \quad GMD = 12.6 \text{ m}$$

$$12.6 = \sqrt[3]{(D)(D)(2D)} \quad \text{or} \quad D = 10 \text{ m}$$

4.8. A three-phase transposed line is composed of one ACSR 159,000 cmil, 54/19 Lapwing conductor per phase with flat horizontal spacing of 8 meters as shown in Figure 37. The GMR of each conductor is 1.515 cm.
(a) Determine the inductance per phase per kilometer of the line.
(b) This line is to be replaced by a two-conductor bundle with 8-m spacing measured from the center of the bundles as shown in Figure 38. The spacing between the conductors in the bundle is 40 cm. If the line inductance per phase is to be 77 percent of the inductance in part (a), what would be the GMR of each new conductor in the bundle?

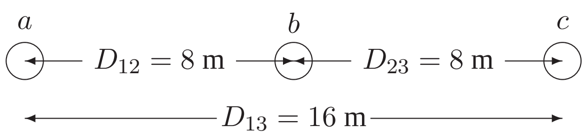


FIGURE 37
Conductor layout for Problem 4.8 (a).

(a)

$$GMD = \sqrt[3]{(8)(8)(16)} = 10.0794 \text{ m}$$

$$L = 0.2 \frac{10.0794}{1.515 \times 10^{-2}} = 1.3 \text{ mH/Km}$$

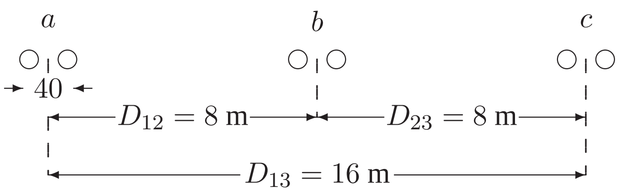


FIGURE 38
Conductor layout for Problem 4.8 (b).

(b)

$$L_2 = (1.3)(0.77) = 1.0 \text{ mH/Km}$$

$$1.0 = 0.2 \ln \frac{10.0794}{D_S^b} \qquad \text{or} \qquad D_S^b = 0.0679 \text{ m}$$

Now

$$S_S^b = \sqrt{D_S\, d} \qquad \text{therefore} \qquad D_S = 1.15 \text{ cm}$$

4.9. A three-phase transposed line is composed of one ACSR 1,431,000 cmil, 47/7 Bobolink conductor per phase with flat horizontal spacing of 11 meters as shown in Figure 39. The conductors have a diameter of 3.625 cm and a GMR of 1.439 cm. The line is to be replaced by a three-conductor bundle of ACSR 477,000 cmil, 26/7 Hawk conductors having the same cross-sectional area of aluminum as the single-conductor line. The conductors have a diameter of 2.1793 cm and a GMR of 0.8839 cm. The new line will also have a flat horizontal configuration, but it is to be operated at a higher voltage and therefore the phase spacing is increased to 14 m as measured from the center of the bundles as shown in Figure 40. The spacing between the conductors in the bundle is 45 cm. Determine
(a) The percentage change in the inductance.
(b) The percentage change in the capacitance.

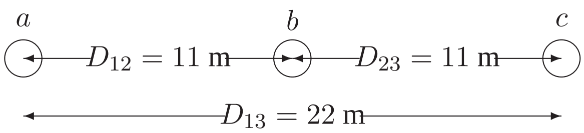


FIGURE 39
Conductor layout for Problem 4.9 (a).

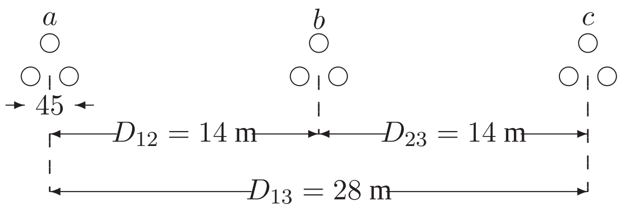


FIGURE 40
Conductor layout for Problem 4.9 (b).

For the one conductor per phase configuration , we have

$$GMD = \sqrt[3]{(11)(11)(22)} = 13.859 \text{ m}$$

$$L = 0.2\frac{13.859}{1.439 \times 10^{-2}} = 1.374 \text{ mH/Km}$$

$$C = \frac{0.0556}{\ln \frac{13.859}{1.8125\times10^{-2}}} = 0.008374 \ \mu\text{F/Km}$$

For the three-conductor bundle per phase configuration , we have

$$GMD = \sqrt[3]{(14)(14)(28)} = 17.6389 \text{ m}$$

$$GMR_L = \sqrt[3]{(45)^2(0.8839)} = 12.1416 \text{ cm}$$

$$GMR_C = \sqrt[3]{(45)^2(2.1793/2)} = 13.01879 \text{ cm}$$

$$L = 0.2\frac{17.6389}{12.1416 \times 10^{-2}} = 0.9957 \text{ mH/Km}$$

$$C = \frac{0.0556}{\ln \frac{17.6389}{13.01879\times10^{-2}}} = 0.011326 \ \mu\text{F/Km}$$

(a) The percentage reduction in the inductance is

$$\frac{1.374 - 0.9957}{1.374}(100) = 27.53\%$$

(b) The percentage increase in the capacitance is

$$\frac{0.001326 - 0.008374}{0.008374}(100) = 35.25\%$$

4.10. A single-circuit three-phase transmission transposed line is composed of four ACSR 1,272,000 cmil conductor per phase with horizontal configuration as shown in Figure 41. The bundle spacing is 45 cm. The conductor code name is *pheasant*. In *MATLAB*, use command **acsr** to find the conductor diameter and its GMR. Determine the inductance and capacitance per phase per kilometer of the line. Use function **[GMD, GMRL, GMRC] =gmd**, (4.58) and (4.92) in *MATLAB* to verify your results.

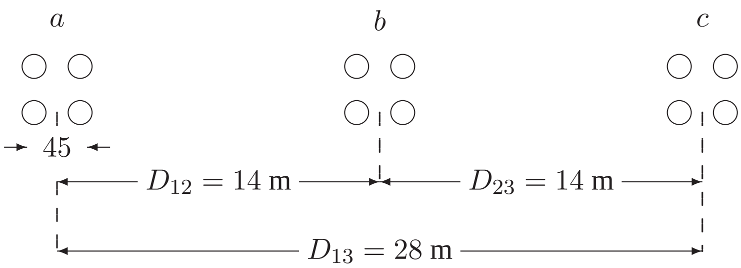


FIGURE 41
Conductor layout for Problem 4.10.

Using the command **acsr**, result in

```
Enter ACSR code name within single quotes -> 'pheasant'
Al Area  Strand  Diameter GMR    Resistance Ohm/Km Ampacity
CMILS    Al/St   cm        cm    60Hz,25C 60HZ,50C Ampere
1272000  54/19  3.5103   1.4173  0.04586  0.05290  1200
```

From (4.79)

$$GMD = \sqrt[3]{(14)(14)(28)} = 17.63889 \text{ m}$$

and From (4.53) and (4.90), we have

$$GMR_L = 1.09\sqrt[4]{(1.4173)(45)^3} = 20.66 \text{ cm}$$

$$GMR_C = 1.09\sqrt[4]{(3.5103/2)(45)^3} = 21.8 \text{ cm}$$

and from (4.58) and (4.92), we get

$$L = 0.2\ln\frac{GMD}{GMRL} = 0.2\ln\frac{17.63889}{0.2066} = 0.889 \text{ mH/Km}$$

and

$$C = \frac{0.0556}{\ln \frac{GMD}{GMRC}} = \frac{0.0556}{\ln \frac{17.63889}{0.218}} = 0.0127 \ \ \mu\text{F/Km}$$

The function **gmd** is used to verify the results. The following commands

```
[GMD, GMRL, GMRC] = gmd
L = 0.2*log(GMD/GMRL);       %mH/Km
C = 0.0556/log(GMD/GMRC);    %microF/Km
```

result in

```
GMD  = 17.63889 m
GMRL =  0.20673 m   GMRC =  0.21808 m
L =
        0.8893
C =
        0.0127
```

4.11. A double circuit three-phase transposed transmission line is composed of two ACSR 2,167,000 cmil, 72/7 Kiwi conductor per phase with vertical configuration as shown in Figure 42. The conductors have a diameter of 4.4069 cm and a GMR of 1.7374 cm. The bundle spacing is 45 cm. The circuit arrangement is $a_1b_1c_1$, $c_2b_2a_2$. Find the inductance and capacitance per phase per kilometer of the line. Find these values when the circuit arrangement is $a_1b_1c_1$, $a_2b_2c_2$. Use function **[GMD, GMRL, GMRC] =gmd**, (4.58) and (4.92) in *MATLAB* to verify your results. For the $a_1b_1c_1$,$c_2b_2a_2$ configuration, the following distances are computed in

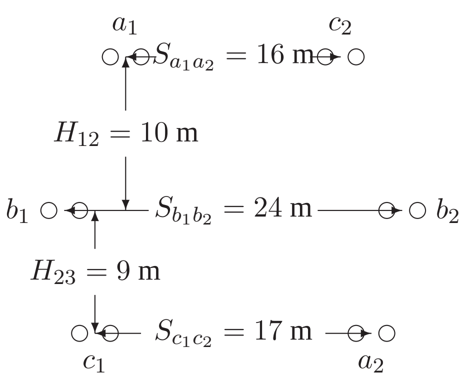


FIGURE 42
Conductor layout for Problem 4.11.

meter.

$$\begin{array}{llll} D_{a_1b_1} = 10.7703 & D_{a_1b_2} = 22.3607 & D_{a_2b_1} = 22.3886 & D_{a_2b_2} = 9.6566 \\ D_{b_1c_1} = 9.6566 & D_{b_1c_2} = 22.3607 & D_{b_2c_1} = 22.3886 & D_{b_2c_2} = 10.7703 \\ D_{a_1c_1} = 19.0065 & D_{a_1c_2} = 16.0000 & D_{a_2c_1} = 17.0000 & D_{a_2c_2} = 19.0065 \\ D_{a_1a_2} = 25.1644 & D_{b_1b_2} = 24.0000 & D_{c_1c_2} = 25.1644 & \end{array}$$

From (4.54), we have

$$D_{AB} = \sqrt[4]{(10.7703)(22.3607)(22.3886)(9.6566)} = 15.1057$$

$$D_{BC} = \sqrt[4]{(9.6566)(22.3607)(22.3886)(10.7703)} = 15.1057$$

$$D_{CA} = \sqrt[4]{(19.0065)(16.0000)(17.0000)(19.0065)} = 17.0749$$

The equivalent geometric mean distance is

$$GMD = \sqrt[3]{(15.1057)(15.1057)(17.7049)} = 15.9267$$

From (4.51) and (4.66), we have

$$D_s^b = \sqrt{D_s \times d} = \sqrt{(0.017374)(0.45)} = 0.08842$$

$$r^b = \sqrt{r \times d} = \sqrt{(0.044069/2)(0.45)} = 0.09957$$

From (4.56) and (4.93), we have

$$\begin{array}{ll} D_{SA} = \sqrt{(0.08842)(25.1644)} = 1.4916 & D_{SB} = \sqrt{(0.08842)(24.0000)} = 1.4567 \\ D_{SC} = \sqrt{(0.08842)(25.1644)} = 1.4916 & \end{array}$$

and

$$\begin{array}{ll} r_A = \sqrt{(0.09957)(25.1644)} = 1.5829 & r_B = \sqrt{(0.09957)(24.0000)} = 1.5458 \\ r_C = \sqrt{(0.09957)(25.1644)} = 1.5829 & \end{array}$$

$$GMR_L = \sqrt[3]{(1.4916)(1.4567)(1.4916)} = 1.4799$$

$$GMR_C = \sqrt[3]{(1.5829)(1.5458)(1.5829)} = 1.5705$$

Therefore, the inductance and capacitance per phase are

$$L = 0.2 \ln \frac{GMD}{GMRL} = 0.2 \ln \frac{15.9267}{1.4799} = 0.4752 \ \text{mH/Km}$$

and

$$C = \frac{0.0556}{\ln \frac{GMD}{GMRC}} = \frac{0.0556}{\ln \frac{15.9267}{1.5705}} = 0.0240 \ \mu\text{F/Km}$$

The function **gmd** is used to verify the results. The following commands

```
[GMD, GMRL, GMRC] = gmd
L = 0.2*log(GMD/GMRL);       %mH/Km
C = 0.0556/log(GMD/GMRC);    %microF/Km
```

result in

```
Circuit Arrangements
--------------------
(1)  abc-c‘b‘a‘
(2)  abc-(a‘)(b‘)(c‘)
Enter (1 or 2)  -> 1
Enter spacing unit within quotes ’m’ or ’ft’-> ’m’
Enter row vector [S11, S22, S33] = [16 24 17]
Enter row vector [H12, H23] = [10 9]
Cond. size, bundle spacing unit: Enter ’cm’ or ’in’-> ’cm’
Conductor diameter in cm = 4.4069
Geometric Mean Radius in cm = 1.7374
No. of bundled cond. (enter 1 for single cond.) = 2
Bundle spacing in cm = 45

GMD  = 15.92670 m
GMRL =  1.47993 m   GMRC =  1.57052 m
L =
        0.4752
C =
        0.0240
```

For the $a_1b_1c_1$,$a_2b_2c_2$ configuration, the following distances are computed in meter.

$$
\begin{array}{llll}
D_{a_1b_1} = 10.7703 & D_{a_1b_2} = 22.3607 & D_{a_2b_1} = 22.3607 & D_{a_2b_2} = 10.7703 \\
D_{b_1c_1} = 9.6566 & D_{b_1c_2} = 22.3886 & D_{b_2c_1} = 22.3886 & D_{b_2c_2} = 9.6566 \\
D_{a_1c_1} = 19.0065 & D_{a_1c_2} = 25.1644 & D_{a_2c_1} = 25.1644 & D_{a_2c_2} = 19.0065 \\
D_{a_1a_2} = 16.0000 & D_{b_1b_2} = 24.0000 & D_{c_1c_2} = 17.0000 &
\end{array}
$$

From (4.54), we have

$$D_{AB} = \sqrt[4]{(10.7703)(22.3607)(22.3607)(10.7703)} = 15.5187$$

$$D_{BC} = \sqrt[4]{(9.6566)(22.3886)(22.3886)(9.6566)} = 14.7036$$

$$D_{CA} = \sqrt[4]{(19.0065)(25.1644)(25.1644)(19.0065)} = 21.8698$$

The equivalent geometric mean distance is

$$GMD = \sqrt[3]{(15.5187)(14.7036)(21.8698)} = 17.0887$$

From (4.51) and (4.66), we have

$$D_s^b = \sqrt{D_s \times d} = \sqrt{(0.017374)(0.45)} = 0.08842$$
$$r^b = \sqrt{r \times d} = \sqrt{(0.044069/2)(0.45)} = 0.09957$$

From (4.56) and (4.93), we have

$$D_{SA} = \sqrt{(0.08842)(16)} = 1.1894 \; D_{SB} = \sqrt{(0.08842)(24.0000)} = 1.4567$$
$$D_{SC} = \sqrt{(0.08842)(17)} = 1.2260$$

and

$$r_A = \sqrt{(0.09957)(16)} = 1.2622 \; r_B = \sqrt{(0.09957)(24.0000)} = 1.5458$$
$$r_C = \sqrt{(0.09957)(17)} = 1.3010$$

$$GMR_L = \sqrt[3]{(1.1894)(1.4567)(1.2260)} = 1.2855$$
$$GMR_C = \sqrt[3]{(1.2622)(1.5458)(1.3010} = 1.3641$$

Therefore, the inductance and capacitance per phase are

$$L = 0.2 \ln \frac{GMD}{GMRL} = 0.2 \ln \frac{17.0887}{1.2855} = 0.5174 \text{ mH/Km}$$

and

$$C = \frac{0.0556}{\ln \frac{GMD}{GMRC}} = \frac{0.0556}{\ln \frac{17.0887}{1.3641}} = 0.02199 \;\; \mu\text{F/Km}$$

The function **gmd** is used in the same way as before to verify the results.

4.12. The conductors of a double-circuit three-phase transmission line are placed on the corner of a hexagon as shown in Figure 43. The two circuits are in parallel and are sharing the balanced load equally. The conductors of the circuits are identical, each having a radius r. Assume that the line is symmetrically transposed. Using the method of GMR, determine an expression for the capacitance per phase per meter of the line.

$$D_{a_1c_1} = D_{a_1b_2} = D_{b_1a_2} = D_{b_1c_2} = D_{c_1b_2} = D_{c_2a_2} = \sqrt{3}D$$

$$D_{AB} = D_{BC} = D_{CA} = \sqrt[4]{D_{a_1b_1}D_{a_1b_2}D_{a_2b_1}D_{a_2b_2}}$$
$$= \sqrt{D_{a_1b_1}D_{a_1b_2}} = 3^{\frac{1}{4}}D$$

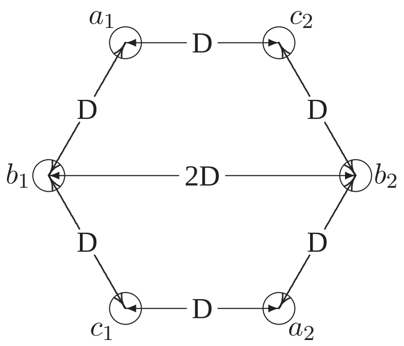


FIGURE 43
Conductor layout for Problem 4.12.

Therefore

$$GMD = 3^{\frac{1}{4}}D$$

$$D_{SA} = D_{SB} = D_{SC} = (r \times 2D)^{\frac{1}{2}}$$

and

$$C = \frac{2\pi\epsilon}{\ln(\frac{3^{\frac{1}{2}}D^2}{2rD})^{\frac{1}{2}}} = \frac{4\pi\epsilon}{\ln\frac{0.866D}{r}} \quad \text{F/m}$$

4.13. A 60-Hz, single-phase power line and a telephone line are parallel to each other as shown in Figure 44. The telephone line is symmetrically positioned directly below phase b. The power line carries an rms current of 226 A. Assume zero current flows in the ungrounded telephone wires. Find the magnitude of the voltage per Km induced in the telephone line.

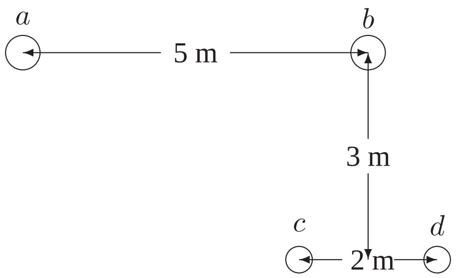


FIGURE 44
Conductor layout for Problem 4.13.

$$D_{ac} = \sqrt{(4)^2 + (3)^2} = 5.0000 \text{ m}$$
$$D_{ad} = \sqrt{(6)^2 + (3)^2} = 6.7082 \text{ m}$$

$$\lambda_{cdI_a} = (0.2)(226) \ln \frac{6.7082}{5.0000} = 13.28 \text{ Wb/Km}$$
$$\lambda_{cdI_b} = 0.2 I_b \ln \frac{3.1622}{3.1622} = 0$$

The total flux linkage is

$$\lambda_{cd} = 13.28 \text{ mWb/Km}$$

The voltage induced in the telephone line per Km is

$$V = \omega \lambda_{cd} = 2\pi 60 (13.28) 10^{-3} = 5 \text{ V/Km}$$

4.14. A three-phase, 60-Hz, untransposed transmission line runs in parallel with a telephone line for 20 km. The power line carries a balanced three-phase rms current of $I_a = 320\angle 0^\circ$ A, $I_b = 320\angle -120^\circ$ A, and $I_c = 320\angle -240^\circ$ A
The line configuration is as shown in Figure 45. Assume zero current flows in the ungrounded telephone wires. Find the magnitude of the voltage induced in the telephone line.

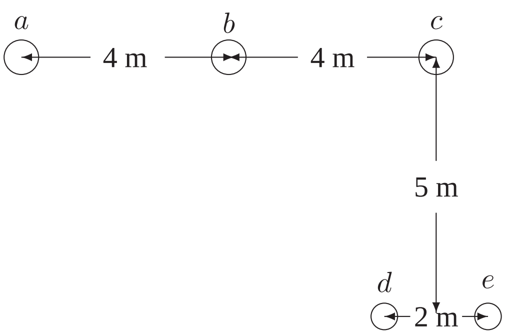


FIGURE 45
Conductor layout for Problem 4.14.

$$D_{ad} = \sqrt{(7)^2 + (5)^2} = 8.6023 \text{ m}$$
$$D_{ae} = \sqrt{(9)^2 + (5)^2} = 10.2956 \text{ m}$$
$$D_{bd} = \sqrt{(3)^2 + (5)^2} = 5.8309 \text{ m}$$
$$D_{be} = \sqrt{(5)^2 + (5)^2} = 7.0711 \text{ m}$$

$$\lambda_{deI_a} = 0.2I_a \ln \frac{10.2956}{8.6023} = 0.03594I_a$$
$$\lambda_{deI_b} = 0.2I_b \ln \frac{7.0711}{5.8309} = 0.03856I_b$$
$$\lambda_{deI_c} = 0.2I_c \ln \frac{5.0249}{5.0249} = 0$$

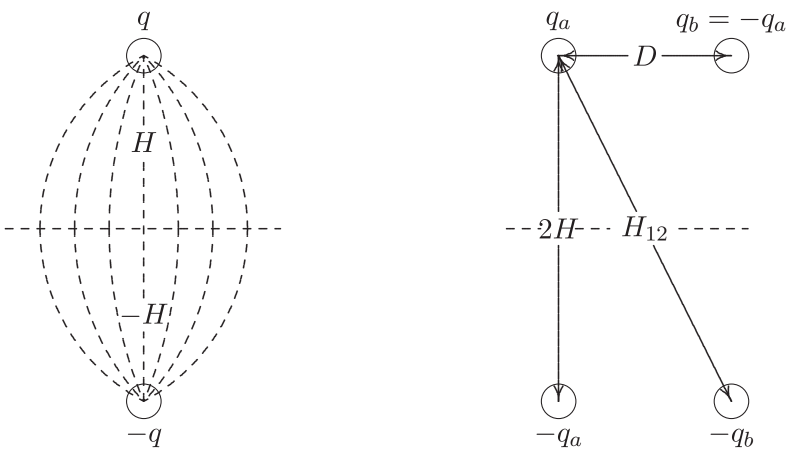
The total flux linkage is

$$\lambda_{de} = (0.03594)320\angle 0^\circ + (0.03856)320\angle{-120^\circ} = 11.943\angle{-63.48^\circ} \quad \text{mWb/Km}$$

The voltage induced in the 20 Km telephone line is

$$V = j\omega\lambda_{de} = j2\pi 60(11.943\angle{-63.48^\circ})(10^{-3})(20) = 90\angle 26.52^\circ \quad \text{V}$$

4.15. Since earth is an equipotential plane, the electric flux lines are forced to cut the surface of the earth orthogonally. The earth effect can be represented by placing an oppositely charged conductor a depth H below the surface of the earth as shown in Figure 4-13(a). This configuration without the presence of the earth will produce the same field as a single charge and the earth surface. This imaginary conductor is called the image conductor. Figure 4-13(b) shows a single-phase line with its image conductors.



(a) Earth plane replaced by image conductor

(b) Single-phase line and its image

FIGURE 46
Conductor layout for Problem 4.15.

Find the potential difference V_{ab} and show that the equivalent capacitance to neutral

is given by

$$C_{an} = C_{bn} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)}$$

$$V_{ab} = \frac{1}{2\pi\epsilon}\left[q_a \ln\frac{D}{r} - q_a \ln\frac{H_{12}}{2H} + q_b \ln\frac{r}{D} - q_b \ln\frac{2H}{H_{12}}\right]$$

Substituting for $q_b = -q_a$, results in

$$V_{ab} = \frac{q_a}{\pi\epsilon}\left[\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)\right]$$

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{\pi\epsilon}{\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)}$$

Therefore, the equivalent capacitance to neutral is

$$C_{an} = C_{bn} = 2C_{ab} = \frac{2\pi\epsilon}{\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)}$$

CHAPTER 5 PROBLEMS

5.1. A 69-kV, three-phase short transmission line is 16 km long. The line has a per phase series impedance of $0.125 + j0.4375\ \Omega$ per km. Determine the sending end voltage, voltage regulation, the sending end power, and the transmission efficiency when the line delivers

(a) 70 MVA, 0.8 lagging power factor at 64 kV.
(b) 120 MW, unity power factor at 64 kV.

Use **lineperf** program to verify your results.

The line impedance is

$$Z = (0.125 + j0.4375)(16) = 2 + j7\ \Omega$$

The receiving end voltage per phase is

$$V_R = \frac{64\angle 0^\circ}{\sqrt{3}} = 36.9504\angle 0^\circ \quad \text{kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 70\angle \cos^{-1} 0.8 = 70\angle 36.87^\circ = 56 + j42 \ \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{70000\angle -36.87^\circ}{3 \times 36.9504\angle 0^\circ} = 631.477\angle -36.87^\circ \quad \text{A}$$

The sending end voltage is

$$\begin{aligned} V_S = V_R + ZI_R &= 36.9504\angle 0^\circ + (2 + j7)(631.477\angle -36.87^\circ)(10^{-3}) \\ &= 40.708\angle 3.9137^\circ \quad \text{kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 70.508 \quad \text{kV}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 40.708\angle 3.9137 \times 631.477\angle 36.87^\circ \times 10^{-3} \\ &= 58.393 \text{ MW} + j50.374 \text{ Mvar} \\ &= 77.1185\angle 40.7837^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{70.508 - 64}{64} \times 100 = 10.169\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{56}{58.393} \times 100 = 95.90\%$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 120\angle 0^\circ = 120 + j0 \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V_R^*} = \frac{120000\angle 0^\circ}{3 \times 36.9504\angle 0^\circ} = 1082.53\angle 0^\circ \quad \text{A}$$

The sending end voltage is

$$\begin{aligned} V_S = V_R + ZI_R &= 36.9504\angle 0^\circ + (2 + j7)(1082.53\angle 0^\circ)(10^{-3}) \\ &= 39.8427\angle 10.9639^\circ \quad \text{kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 69.0096 \quad \text{kV}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 39.8427\angle 10.9639 \times 1082.53\angle 0^\circ \times 10^{-3} \\ &= 127.031 \text{ MW} + j24.609 \text{ Mvar} \\ &= 129.393\angle 10.9639^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{69.0096 - 64}{64} \times 100 = 7.8275\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{120}{127.031} \times 100 = 94.465\%$$

The above computations are performed efficiently using the **lineperf** program.

The command:

```
lineperf
```

displays the following menu

```
Type of parameters for input                Select
Parameters per unit length
r (Ω), g (Siemens), L (mH), C (μF)             1

Complex z and y per unit length
r + j*x (Ω), g + j*b (Siemens)                 2

Nominal π or Eq. π model                       3

A, B, C, D constants                           4

Conductor configuration and dimension          5

To quit                                        0
Select number of menu → 2
Enter line length = 16
Enter frequency in Hz = 60
Enter series impedance r + j*x in ohm per unit length
z = 0.125+j*0.4375
Enter shunt admittance g + j*b in siemens per unit length
y = 0+j*0
```

```
Short line model
Z = 2 + j 7 ohms
```

$$\text{ABCD} = \begin{bmatrix} 1 & + & j0 & 2 & + & j7 \\ 0 & + & j0 & 1 & + & j0 \end{bmatrix}$$

```
      Transmission line performance
             Analysis                                Select

To calculate sending end quantities
for specified receiving end MW, Mvar                   1

To calculate receiving end quantities
for specified sending end MW, Mvar                     2

To calculate sending end quantities
when load impedance is specified                       3

Open-end line and reactive compensation                4

Short-circuited line                                   5

Capacitive compensation                                6

Receiving end circle diagram                           7

Loadability curve and voltage profile                  8

To quit                                                0

Select number of menu → 1

Enter receiving end line-line voltage kV = 64
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 56
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 42

Line performance for specified receiving end quantities

Vr = 64 kV (L-L) at 0°
Pr = 56 MW Qr = 42 Mvar
```

```
Ir = 631.477 A at -36.8699° PFr = 0.8 lagging
Vs = 70.5082 kV (L-L) at 3.91374°
Is = 631.477 A at -36.8699° PFs = 0.757182 lagging
Ps = 58.393 MW     Qs = 50.374 Mvar
PL = 2.393 MW     QL = 8.374 Mvar
Percent Voltage Regulation = 10.169
Transmission line efficiency = 95.9026
```

At the end of this analysis the **listmenu** (Analysis Menu) is displayed. Selecting option 1 and entering data for part (b), will result in

```
Line performance for specified receiving end quantities
-------------------------------------------------------

Vr = 64 kV (L-L) at 0°
Pr = 120 MW Qr = 0 Mvar
Ir = 1082.53 A at 0° PFr = 1
Vs = 69.0096 kV (L-L) at 10.9639°
Is = 1082.53 A at 0° PFs = 0.981747
Ps = 127.031 MW     Qs = 24.069 Mvar
PL = 7.031 MW     QL = 24.069 Mvar
Percent Voltage Regulation = 7.82754
Transmission line efficiency = 94.4649
```

5.2. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.1. The line delivers 70 MVA, 0.8 lagging power factor at 64 kV. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is

(a) 69 kV.
(b) 64 kV.
Hint: Use (5.85) and (5.86) to compute the power angle δ and the receiving end reactive power.
(c) Use **lineperf** to obtain the compensated line performance.

The complex load at the receiving end is

$$S_{R(3\phi)} = 70\angle\cos^{-1}0.8 = 56 \text{ MW} + j42 \text{ Mvar}$$

$$Z = 2 + j7 = 7.28\angle 74.0546^\circ$$

(a) For $V_{R(LL)} = 69$ kV, from (5.85), we have

$$56 = \frac{(69)(64)}{7.28}\cos(74.0564 - \delta) - \frac{(1.0)(64)^2}{7.28}\cos 74.0546^\circ$$

Therefore,

$$\cos(74.0546 - \delta) = 0.3471 \qquad \text{or} \qquad \delta = 4.3646$$

Now from (5.86), we have

$$Q_{R(3\phi)} = \frac{(69)(64)}{7.28}\sin(74.0564 - 4.3646) - \frac{(1.0)(64)^2}{7.28}\sin 74.0546^\circ$$
$$= 27.883 \ \text{Mvar}$$

Therefore, the required capacitor Mvar is

$$Q_C = 42 - 27.883 = 14.117 \ \text{Mvar}$$

$$Z_C = \frac{|V_R|^2}{S_C^*} = \frac{(64/sqrt3)^2}{j14.117/3} = -j290.147 \ \Omega$$

The required shunt capacitance per phase is

$$C = \frac{10^6}{(2\pi)(60)(290.147)} = 9.1422 \ \mu\text{F}$$

(b) For $V_{R(LL)} = 64$ kV, from (5.85), we have

$$56 = \frac{(64)(64)}{7.28}\cos(74.0564 - \delta) - \frac{(1.0)(64)^2}{7.28}\cos 74.0546^\circ$$

Therefore,

$$\cos(74.0546 - \delta) = 0.37425 \qquad \text{or} \qquad \delta = 6.0327^\circ$$

Now from (5.86), we have

$$Q_{R(3\phi)} = \frac{(64)(64)}{7.28}\sin(74.0564 - 6.0327) - \frac{(1.0)(64)^2}{7.28}\sin 74.0546^\circ$$
$$= \ -19.2405 \ \text{Mvar}$$

Therefore, the required capacitor Mvar is

$$Q_C = 42 - (-19.2405) = 61.2405 \ \text{Mvar}$$

$$Z_C = \frac{|V_R|^2}{S_C^*} = \frac{(64/sqrt3)^2}{j61.2405/3} = -j66.8838 \ \Omega$$

The required shunt capacitance per phase is

$$C = \frac{10^6}{(2\pi)(60)(66.8838)} = 39.6596 \ \mu\text{F}$$

With **lineperf** program, we continue the analysis in Problem 5.1 by selecting option 6. This will display the **compmenu**

```
        Capacitive compensation
              Analysis                        Select

Shunt capacitive compensation                   1

Series capacitive compensation                  2

Series and shunt capacitive compensation        3

To quit                                         0
```

Selecting option 1 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 69
Enter desired receiving end line-line voltage kV = 64
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 56
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 42

Shunt capacitive compensation

Vs = 69 kV (L-L) at 4.36672°
Vr = 64 kV (L-L) at 0°
Pload = 56 MW,     Qload = 42 Mvar
Load current = 631.477 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 290.147 Ω, 9.14222 μF, 14.117 Mvar
Shunt capacitor current = 127.351 A at 90°
Pr = 56.000 MW,     Qr = 27.883 Mvar
Ir = 564.339 A at -26.4692°, PFr = 0.895174 lagging
Is = 564.339 A at -26.4692° PFs = 0.858639 lagging
Ps = 57.911 MW,     Qs = 34.571 Mvar
PL = 1.911 MW,     QL = 6.688 Mvar
Percent Voltage Regulation = 7.8125
Transmission line efficiency = 96.7003
```

Repeating for part (b), we have

```
Shunt capacitive compensation

Vs = 64 KV (L-L) at 6.03281°
Vr = 64 KV (L-L) at 0°
Pload = 56 MW,     Qload = 42 Mvar
Load current = 631.477 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 66.8837Ω, 39.6596 μF, 61.2406 Mvar
```

```
Shunt capacitor current = 552.457 A at 90°
Pr = 56.000 MW,     Qr = -19.241 Mvar
Ir = 534.168 A at 18.9618°, PFr = 0.945735 leading
Is = 534.168 A at 18.9618° PFs = 0.974648 leading
Ps = 57.712 MW,     Qs = -13.249 Mvar
PL = 1.712 MW,     QL = 5.992 Mvar
Percent Voltage Regulation = 2.22045e-14
Transmission line efficiency = 97.0335
```

5.3. A 230-kV, three-phase transmission line has a per phase series impedance of $z = 0.05 + j0.45\ \Omega$ per Km and a per phase shunt admittance of $y = j3.4 \times 10^{-6}$ siemens per km. The line is 80 km long. Using the nominal π model, determine

(a) The transmission line ABCD constants. Find the sending end voltage and current, voltage regulation, the sending end power and the transmission efficiency when the line delivers
(b) 200 MVA, 0.8 lagging power factor at 220 kV.
(c) 306 MW, unity power factor at 220 kV.

Use **lineperf** program to verify your results.

The line impedance and shunt admittance are

$$Z = (0.05 + j0.45)(80) = 4 + j36\ \Omega$$

$$Y = (j3.4 \times 10^{-6})(80) = j0.272 \times 10^{-3} \text{ siemens}$$

The ABCD constants of the nominal π model are

$$A = (1 + \frac{ZY}{2}) = (1 + \frac{(4 + j36)(j0.272 \times 10^{-3})}{2}) = 0.9951 + j0.000544$$
$$B = Z = 4 + j36$$
$$C = Y(1 + \frac{ZY}{4}) = j0.0002713$$

The receiving end voltage per phase is

$$V_R = \frac{220\angle 0^\circ}{\sqrt{3}} = 127\angle 0^\circ \ \text{ kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 200\angle \cos^{-1} 0.8 = 200\angle 36.87^\circ = 160 + j120 \ \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{200000\angle - 36.87^\circ}{3 \times 127\angle 0^\circ} = 524.864\angle - 36.87^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.9951 + j0.000544)(127\angle 0^\circ) + (4 + j36)$$
$$(524.864 \times 10^{-3}\angle{-36.87^\circ}) = 140.1051\angle 5.704^\circ \quad \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 242.67 \quad \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0002713)(127000\angle 0^\circ) + (0.9951 + j0.000544)$$
$$(524.864\angle{-36.87^\circ}) = 502.38\angle{-33.69^\circ} \quad \text{A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I^*_S &= 3 \times 140.1051\angle 5.704 \times 502.38\angle 33.69^\circ \times 10^{-3} \\ &= 163.179 \text{ MW} + j134.018 \text{ Mvar} \\ &= 211.16\angle 39.396^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{242.67}{0.9951} - 220}{220} \times 100 = 10.847\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{160}{163.179} \times 100 = 98.052\%$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 306\angle 0^\circ = 306 + j0 \;\; \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{306000\angle 0^\circ}{3 \times 127\angle 0^\circ} = 803.402\angle 0^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.9951 + j0.000544)(127\angle 0^\circ) + (4 + j36)$$
$$(803.402 \times 10^{-3}\angle 0^\circ) = 132.807\angle 12.6^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 230.029 \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0002713)(127000\angle 0^\circ) + (0.9951 + j0.000544)$$
$$(803.402\angle 0^\circ) = 799.862\angle 2.5^\circ \text{ A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 132.807\angle 12.6 \times 799.862\angle -2.5^\circ \times 10^{-3} \\ &= 313.742 \text{ MW} + j55.9 \text{ Mvar} \\ &= 318.68\angle 10.1^\circ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{230.029}{0.9951} - 220}{220} \times 100 = 5.073\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{306}{313.742} \times 100 = 97.53\%$$

The above computations are performed efficiently using the **lineperf** program. The command:

```
lineperf
```

displays the following menu

```
Type of parameters for input                      Select
Parameters per unit length
r (Ω), g (Siemens), L (mH), C (μF)                  1

Complex z and y per unit length
r + j*x (Ω), g + j*b (Siemens)                      2
```

```
Nominal π or Eq. π model                         3

A, B, C, D constants                             4

Conductor configuration and dimension            5

To quit                                          0
Select number of menu → 2
Enter line length = 80
Enter frequency in Hz = 60
Enter series impedance r + j*x in ohm per unit length z =
0.05 + j0.45
Enter shunt admittance g + j*b in siemens per unit length y =
0+j*3.4e-6

Short line model
Z = 4 + j 36 ohms
Y = 0 + j 0.000272 Siemens
```

$$\mathtt{ABCD} = \begin{bmatrix} 0.9951 & + & j0.000544 & 4 & + & j36 \\ -7.3984e-008 & + & j0.00027133 & 0.9951 & + & j0.000544 \end{bmatrix}$$

```
      Transmission line performance
             Analysis                                Select

To calculate sending end quantities
for specified receiving end MW, Mvar                   1

To calculate receiving end quantities
for specified sending end MW, Mvar                     2

To calculate sending end quantities
when load impedance is specified                       3

Open-end line and reactive compensation                4

Short-circuited line                                   5

Capacitive compensation                                6
```

```
Receiving end circle diagram                          7

Loadability curve and voltage profile                 8

To quit                                               0

Select number of menu → 1

Enter receiving end line-line voltage kV = 220
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 160
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 120

Line performance for specified receiving end quantities
-------------------------------------------------------

Vr = 220 kV (L-L) at 0°
Pr = 160 MW Qr = 120 Mvar
Ir = 524.864 A at -36.8699° PFr = 0.8 lagging
Vs = 242.67 kV (L-L) at 5.7042°
Is = 502.381 A at -33.6919° PFs = 0.77277 lagging
Ps = 163.179 MW    Qs = 134.018 Mvar
PL = 3.179 MW    QL = 14.018 Mvar
Percent Voltage Regulation = 10.847
Transmission line efficiency = 98.052
```

At the end of this analysis the **listmenu** (Analysis Menu) is displayed. Selecting option 1 and entering data for part (b), will result in

```
Line performance for specified receiving end quantities
-------------------------------------------------------

Vr = 220 kV (L-L) at 0°
Pr = 306 MW Qr = 0 Mvar
Ir = 803.042 A at 0° PFr = 1
Vs = 230.029 kV (L-L) at 12.6033°
Is = 799.862 A at 2.5008° PFs = 0.984495
Ps = 313.742 MW    Qs = 55.900 Mvar
PL = 7.742 MW    QL = 55.900 Mvar
Percent Voltage Regulation = 5.0732
Transmission line efficiency = 97.532
```

5.4. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.3. The line delivers 200 MVA, 0.8 lagging power factor at 220 kV.

(a) Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is 220 kV. *Hint*: Use (5.85) and (5.86) to compute the power angle δ and the receiving end reactive power.
(b) Use **lineperf** to obtain the compensated line performance.

(a) The complex load at the receiving end is

$$S_{R(3\phi)} = 200\angle 36.87^\circ = 160 \text{ MW} + j120 \text{ Mvar}$$

$$Z = 4 + j36 = 36.2215\angle 83.6598^\circ$$

(a) For $V_{R(LL)} = 220$ kV, from (5.85), we have

$$160 = \frac{(220)(220)}{36.2215}\cos(83.6598^\circ - \delta) - \frac{(0.9951)(220)^2}{36.2215}\cos(83.6598^\circ - 0.0313^\circ)$$

Therefore,

$$\cos(83.6598 - \delta) = 0.23017$$

or

$$\delta = 6.967^\circ$$

Now from (5.86), we have

$$Q_{R(3\phi)} = \frac{(220)(220)}{36.2215}\sin(83.6598^\circ - 6.967^\circ) - \frac{(0.9951)(220)^2}{36.2215}$$
$$\sin(83.6598^\circ - 6.967)^\circ = -21.12 \ \text{Mvar}$$

Therefore, the required capacitor Mvar is

$$Q_C = 21.12 + 120 = 141.12 \ \text{Mvar}$$

$$Z_C = \frac{|V_R|^2}{S_C^*} = \frac{(220/sqrt3)^2}{j141.12/3} = -j342.964\ \Omega$$

The required shunt capacitance per phase is

$$C = \frac{10^6}{(2\pi)(60)(342.964)} = 141.123 \ \ \mu\text{F}$$

With **lineperf** program, we continue the analysis in Problem 5.1 by selecting option 6. This will display the **compmenu**

```
        Capacitive compensation
              Analysis                      Select

Shunt capacitive compensation                 1

Series capacitive compensation                2

Series and shunt capacitive compensation      3

To quit                                       0
```

Selecting option 1 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 220
Enter desired receiving end line-line voltage kV = 220
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 160
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 120

Shunt capacitive compensation

Vs = 220 kV (L-L) at 6.96702°
Vr = 220 kV (L-L) at 0°
Pload = 160 MW,    Qload = 120 Mvar
Load current = 524.864 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 342.964 Ω, 7.73428 μF, 141.123 Mvar
Shunt capacitor current = 370.351 A at 90°
Pr = 160.000 MW,    Qr = -21.123 Mvar
Ir = 423.534 A at 7.52047°, PFr = 0.991398 leading
Is = 427.349 A at 12.1375° PFs = 0.995931 leading
Ps = 162.179 MW,    Qs = -14.675 Mvar
PL = 2.179 MW,    QL = 6.447 Mvar
Percent Voltage Regulation = 0.49199
Transmission line efficiency = 98.6563
```

5.5. A three-phase, 345-kV, 60-Hz transposed line is composed of two ACSR 1,113,000, 45/7 Bluejay conductors per phase with flat horizontal spacing of 11 m. The conductors have a diameter of 3.195 cm and a GMR of 1.268 cm. The bundle spacing is 45 cm. The resistance of each conductor in the bundle is $0.0538\ \Omega$ per km and the line conductance is negligible. The line is 150 Km long. Using the nominal π model, determine the ABCD constant of the line. Use **lineperf** and option 5 to verify your results.

$$GMD = \sqrt[3]{(11)(11)(22)} = 13.859 \text{ m}$$

$$GMR_L = \sqrt{(45)(1.268)} = 7.5538 \ \text{ cm}$$

$$GMR_C = \sqrt{(45)(3.195/2)} = 8.47865 \ \text{ cm}$$

$$L = 0.2\frac{13.859}{7.5538 \times 10^{-2}} = 1.0424 \ \text{ mH/Km}$$

$$C = \frac{0.0556}{\ln \frac{13.859}{8.47865 \times 10^{-2}}} = 0.010909 \ \mu\text{F/Km}$$

$$Z = (\frac{0.0538}{2} + j2\pi \times 60 \times 1.0424 \times 10^{-3})(150) = 4.035 + j58.947$$

$$Y = j(2\pi 60 \times 0.9109 \times 10^{-6})(150) = j0.0006169$$

The ABCD constants of the nominal π model are

$$\begin{aligned} A &= \left(1 + \frac{ZY}{2}\right) = \left(1 + \frac{(4.035 + j38.947)(j0.0006169)}{2}\right) \\ &= 0.98182 + j0.0012447 \\ B &= Z = 4.035 + j58.947 \\ C &= Y\left(1 + \frac{ZY}{4}\right) = j0.00061137 \end{aligned}$$

Using **lineperf** and option 5, result in

Type of parameters for input	Select
Parameters per unit length r (Ω), g (Siemens), L (mH), C (μF)	1
Complex z and y per unit length r + j*x (Ω), g + j*b (Siemens)	2
Nominal π or Eq. π model	3
A, B, C, D constants	4
Conductor configuration and dimension	5
To quit	0

Select number of menu → 5

When the line configuration and conductor specifications are entered, the following results are obtained

```
GMD = 13.85913 m
GMRL = 0.07554 m       GMRC = 0.08479 m
L = 1.04241 mH/Km       C = 0.0109105 micro F/Km

Short line model
Z = 4.035 + j 58.947 ohms
Y = 0 + j 0.000616976 Siemens
```

$$\text{ABCD} = \begin{bmatrix} 0.98182 & + & j0.0012447 & 4.035 & + & j58.947 \\ -3.8399e-007 & + & j0.00061137 & 0.98182 & + & j0.0012447 \end{bmatrix}$$

5.6. The ABCD constants of a three-phase, 345-kV transmission line are

$$A = D = 0.98182 + j0.0012447$$
$$B = 4.035 + j58.947$$
$$C = j0.00061137$$

The line delivers 400 MVA at 0.8 lagging power factor at 345 kV. Determine the sending end quantities, voltage regulation, and transmission efficiency.

The receiving end voltage per phase is

$$V_R = \frac{345\angle 0^\circ}{\sqrt{3}} = 199.186\angle 0^\circ \quad \text{kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 400\angle \cos^{-1} 0.8 = 400\angle 36.87^\circ = 320 + j240 \;\; \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{400000\angle -36.87^\circ}{3 \times 199.186\angle 0^\circ} = 669.392\angle -36.87^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.98182 + j0.0012447)(199.186\angle 0^\circ) + (4.035 + j58.947)$$
$$(668.392 \times 10^{-3}\angle{-36.87^\circ}) = 223.449\angle 7.766^\circ \quad \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 387.025 \;\; \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.00061137)(199.186\angle 0^\circ) + (0.98182 + j0.0012447)$$
$$(669.392\angle{-36.87^\circ}) = 592.291\angle{-27.3256^\circ} \;\; \text{A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 223.449\angle 7.766 \times 592.291\angle 27.3256^\circ \times 10^{-3}$$
$$= 324.872 \text{ MW} + j228.253 \text{ Mvar}$$
$$= 397.041\angle 35.0916^\circ \text{ MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{387.025}{0.98182} - 345}{345} \times 100 = 14.2589\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{320}{324.872} \times 100 = 98.500\%$$

Using **lineperf** and option 4, result in

```
Select number of menu → 4
Enter the complex constant A = 0.98182+j*0.0012447
Enter the complex constant B = 4.035+j*58.947
Enter the complex constant C = j*0.00061137
Two port model, ABCD constants
Z = 4.035 + j 58.947 ohm
Y = 3.87499e-007 + j 0.000616978 siemens
```

$$\texttt{ABCD} = \begin{bmatrix} 0.98182 & + & j0.0012447 & 4.035 & + & j58.947 \\ 0 & + & j0.00061137 & 0.98182 & + & j0.0012447 \end{bmatrix}$$

```
        Transmission line performance
               Analysis                              Select

To calculate sending end quantities
for specified receiving end MW, Mvar                   1
```

```
To calculate receiving end quantities
for specified sending end MW, Mvar                    2

To calculate sending end quantities
when load impedance is specified                      3

Open-end line and reactive compensation               4

Short-circuited line                                  5

Capacitive compensation                               6

Receiving end circle diagram                          7

Loadability curve and voltage profile                 8

To quit                                               0

Select number of menu → 1

Enter receiving end line-line voltage kV = 345
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 320
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 240

Line performance for specified receiving end quantities
-------------------------------------------------------

Vr = 345 kV (L-L) at 0°
Pr = 320 MW Qr = 240 Mvar
Ir = 669.392 A at -36.8699° PFr = 0.8 lagging
Vs = 387.025 kV (L-L) at 7.76603°
Is = 592.291 A at -27.3222° PFs = 0.818268 lagging
Ps = 324.872 MW    Qs = 228.253 Mvar
PL = 4.872 MW    QL = -11.747 Mvar
Percent Voltage Regulation = 14.2589
Transmission line efficiency = 98.5002
```

5.7. Write a *MATLAB* function named **[ABCD] = abcdm(z, y, Lngt)** to evaluate and return the ABCD transmission matrix for a medium-length transmission line where **z** is the per phase series impedance per unit length, **y** is the shunt admittance per unit length, and **Lngt** is the line length. Then, write a program that uses the above function and computes the receiving end quantities, voltage regulation, and transmission efficiency when sending end quantities are specified. The program

should prompt the user to enter the following quantities:

The sending end line-to-line voltage magnitude in kV
The sending end voltage phase angle in degrees
The three-phase sending end real power in MW
The three-phase sending end reactive power in Mvar

Use your program to obtain the solution for the following case:

A three-phase transmission line has a per phase series impedance of $z = 0.03 + j0.4\ \Omega$ per Km and a per phase shunt admittance of $y = j4.0 \times 10^{-6}$ siemens per Km. The line is 125 Km long. Obtain the ABCD transmission matrix. Determine the receiving end quantities, voltage regulation, and the transmission efficiency when the line is sending 407 MW, 7.833 Mvar at 350 kV.

The following function named **abcdm** returns the line ABCD constants

```
function [ABCD] = abcdm(z, y, Lngt);
Z = z*Lngt;
Y = y*Lngt;
A = 1 + Z*Y/2;  B = Z;
C = Y*(1 + Z*Y/4);  D = A;
ABCD = [A B; C D];
```

The following program saved as **ch5p7.m** computes the receiving end quantities from the specified sending end quantities.

```
z=input('Line series impedance per phase per unit length z=');
y=input('Line shunt admittance per phase per unit length y=');
Lngt = input('Transmission line length = ');
ABCD = abcdm(z, y,Lngt)
VL_s=input('Sending end line-to-line voltage magnitude in kV=');
AngV_s=input('Sending end voltage phase angle in degree = ');
P_s =input('Three-phase sending end real power in MW ');
Q_s =input('Three-phase sending end reactive power in Mvar ');
S_s = P_s + j*Q_s;          % MVA
AngV_srd = AngV_s*pi/180;  % Radian
V_s = VL_s/sqrt(3)*(cos(AngV_srd) + j*sin(AngV_srd));      %kV
I_s = conj(S_s)/(3*conj(V_s));   % kA
IL_s= abs(I_s)*1000; AngI_srd = angle(I_s);
AngI_s = AngI_srd*180/pi;
VI_r = inv(ABCD)*[V_s; I_s];
V_r = VI_r(1);  VL_r = sqrt(3)*abs(V_r);
AngV_rrd = angle(V_r); AngV_r = AngV_rrd*180/pi;
```

```
I_r =  VI_r(2);  IL_r = abs(I_r)*1000; AngI_rd = angle(I_r);
AngI_r = AngI_rd*180/pi;
S_r = 3*V_r*conj(I_r); P_r = real(S_r);
Q_r = imag(S_r); A = abs(ABCD(1,1));
Reg = (VL_s/A - VL_r)/VL_r*100;
Eff = P_r/P_s*100;
fprintf('Sending end line-to-line voltage =%g KV\n', VL_s)
fprintf('Sending end voltage phase angle =%g Degree\n',AngV_s)
fprintf('Sending end real power =  %g MW\n', P_s)
fprintf('Sending end reactive Power = %g Mvar\n', Q_s)
fprintf('Sending end current  =  %g A\n', IL_s)
fprintf('Sending end current phase angle=%g Degree \n',AngI_s)
fprintf('receiving end line-to-line voltage =  %g KV\n',VL_r)
fprintf('receiving end voltage phase angle=%g Degree\n',AngV_r)
fprintf('receiving end real power =  %g MW\n', P_r)
fprintf('receiving end reactive Power =  %g Mvar\n', Q_r)
fprintf('receiving end current  =  %g A\n', IL_r)
fprintf('receiving end current phase angle=%g Degree\n',AngI_r)
fprintf('Voltage regulation = %g percent \n',Reg)
fprintf('Transmission efficiency = %g percent \n',Eff)
```

typing **ch5p7** at the *MATLAB* prompt result in

```
Line series impedance per phase per unit length z = 0.03+j*0.4
Line shunt admittance per phase per unit length y = j*4.0e-6
Transmission line length = 125

    ABCD =

       0.9875+ 0.0009i   3.7500+50.0000i
       0.0000+ 0.0005i   0.9875+ 0.0009i

    Sending end line-to-line voltage magnitude in kV = 350
    Sending end voltage phase angle in degree = 0
    Three-phase sending end real power in MW 407
    Three-phase sending end reactive power in Mvar 7.883
    Sending end line-to-line voltage =  350 kV
    Sending end voltage phase angle  =  0 Degree
    Sending end real power =  407 MW
    Sending end reactive Power =  7.883 Mvar
    Sending end current  =  671.502 A
    Sending end current phase angle =  -1.1096 Degree
    Receiving end line-to-line voltage =  345.003 kV
    Receiving end voltage phase angle  =  -9.63278 Degree
    Receiving end real power =  401.884 MW
```

```
Receiving end reactive Power =  0.0475969 Mvar
Receiving end current  =  672.539 A
Receiving end current phase angle =  -9.63957 Degree
Voltage regulation = 2.73265 percent
Transmission efficiency = 98.7429 percent
```

5.8. Obtain the solution for Problems 5.8 through 5.13 using the **lineperf** program. Then, solve each Problem using hand calculations.

A three-phase, 765-kV, 60-Hz transposed line is composed of four ACSR 1,431,000, 45/7 Bobolink conductors per phase with flat horizontal spacing of 14 m. The conductors have a diameter of 3.625 cm and a GMR of 1.439 cm. The bundle spacing is 45 cm. The line is 400 Km long, and for the purpose of this problem, a lossless line is assumed.

(a) Determine the transmission line surge impedance Z_c, phase constant β, wavelength λ, the surge impedance loading SIL, and the ABCD constant.
(b) The line delivers 2000 MVA at 0.8 lagging power factor at 735 kV. Determine the sending end quantities and voltage regulation.
(c) Determine the receiving end quantities when 1920 MW and 600 Mvar are being transmitted at 765 kV at the sending end.
(d) The line is terminated in a purely resistive load. Determine the sending end quantities and voltage regulation when the receiving end load resistance is $264.5\ \Omega$ at 735 kV.

Use the command **lineperf** to obtain the solution for problem 5.8 through 5.13.

(a) For hand calculation we have

$$GMD = \sqrt[3]{(14)(14)(28)} = 17.6389 \ \text{ m}$$

$$GMR_L = 1.09\sqrt[4]{(45)^3(1.439)} = 20.75 \ \text{ cm}$$

$$GMR_C = 1.09\sqrt[4]{(45)^3(3.625/2)} = 21.98 \ \text{ cm}$$

$$L = 0.2\frac{17.6389}{20.75 \times 10^{-2}} = 0.88853 \ \text{ mH/Km}$$

$$C = \frac{0.0556}{\ln\frac{17.6389}{21.98\times 10^{-2}}} = 0.01268 \ \ \mu\text{F/Km}$$

$$\beta = \omega\sqrt{LC} = 2\pi \times 60\,\sqrt{0.88853 \times 0.01268 \times 10^{-9}} = 0.001265 \ \text{ Radian/Km}$$

$$\beta\ell = (0.001265 \times 400)(180/\pi) = 29^\circ$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.001265} = 4967 \ \text{ Km}$$

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.88853 \times 10^{-3}}{0.01268 \times 10^{-6}}} = 264.7 \ \ \Omega$$

$$SIL = \frac{(KV_{Lrated})^2}{Z_c} = \frac{(765)^2}{264.7} = 2210.89 \ \text{ MW}$$

The $ABCD$ constants of the line are

$$\begin{aligned}
A &= \cos\beta\ell = cos29^\circ = 0.8746\\
B &= jZ_c \sin\beta\ell = j264.7\sin 29^\circ = j128.33\\
C &= j\frac{1}{Z_c}\sin\beta\ell = j\frac{1}{264.7}\sin 29^\circ = j0.0018315\\
D &= A
\end{aligned}$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 2000\angle 36.87^\circ = 1600 \text{ MW} + j1200 \text{ Mvar}$$

$$V_R = \frac{735\angle 0^\circ}{\sqrt{3}} = 424.352\angle 0^\circ \ \text{ kV}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V_R^*} = \frac{2000000\angle - 36.87^\circ}{3 \times 424.352\angle 0^\circ} = 1571.02\angle - 36.87^\circ \ \ \text{A}$$

The sending end voltage is

$$\begin{aligned}
V_S = AV_R + BI_R &= (0.8746)(424.352\angle 0^\circ) + (j128.33)\\
&\quad (1571.02 \times 10^{-3}\angle{-36.87^\circ}) = 517.86\angle 18.147^\circ \ \text{ kV}
\end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 896.96 \ \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0018315)(424352\angle 0^\circ) + (0.8746)$$
$$(1571.02\angle{-36.87^\circ}) = 1100.23\angle{-2.46^\circ} \text{ A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 517.86\angle 18.147 \times 1100.23\angle 2.46^\circ \times 10^{-3} \\ &= 1600 \text{ MW} + j601.59 \text{ Mvar} \\ &= 1709.3\angle 20.6^\circ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{896.96}{0.8746} - 735}{735} \times 100 = 39.53\%$$

(c) The complex power at the sending end is

$$S_{S(3\phi)} = 1920 \text{ MW} + j600 \text{ Mvar} = 2011.566\angle 17.354^\circ \text{ MVA}$$

$$V_S = \frac{765\angle 0^\circ}{\sqrt{3}} = 441.673\angle 0^\circ \text{ kV}$$

The sending end current per phase is given by

$$I_S = \frac{S^*_{S(3\phi)}}{3\,V_S^*} = \frac{2011566\angle - 17.354^\circ}{3 \times 441.673\angle 0^\circ} = 1518.14\angle - 17.354^\circ \text{ A}$$

The receiving end voltage is

$$V_R = DV_S - BI_S = (0.8746)(441.673\angle 0^\circ) - (j128.33)$$
$$(1518.14 \times 10^{-3}\angle{-17.354^\circ}) = 377.2\angle{-29.537^\circ} \text{ kV}$$

The receiving end line-to-line voltage magnitude is

$$|V_{R(L-L)}| = \sqrt{3}\,|V_S| = 653.33 \text{ kV}$$

The receiving end current is

$$I_R = -CV_S + AI_S = (-j0.0018315)(441673\angle 0^\circ) + (0.8746)$$
$$(1518.14\angle{-17.354^\circ}) = 1748.73\angle{-43.55^\circ} \text{ A}$$

The receiving end power is

$$S_{R(3\phi)} = 3V_R I_R^* = 3 \times 377.2\angle -29.537^\circ \times 1748.73\angle 43.55^\circ \times 10^{-3}$$
$$= 1920 \text{ MW} + j479.2 \text{ Mvar}$$
$$= 1978.86\angle 14.013^\circ \quad \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{765}{0.8746} - 653.33}{653.33} \times 100 = 33.88\%$$

(d)

$$V_R = \frac{735\angle 0^\circ}{\sqrt{3}} = 424.352\angle 0^\circ \ \text{ kV}$$

The receiving end current per phase is given by

$$I_R = \frac{V_R}{Z_L} = \frac{424352\angle 0^\circ}{264.5} = 1604.357\angle 0^\circ \ \text{ A}$$

The complex power at the receiving end is

$$S_{R(3\phi)} = 3V_R I_R{}^* = 3(424.352\angle 0^\circ)(1604.357\angle 0^\circ) \times 10^{-3} = 2042.44 \ \text{ MW}$$

The sending end voltage is

$$V_S = AV_R + BI_R = (0.8746)(424.352\angle 0^\circ) + (j128.33)$$
$$= (1604.357 \times 10^{-3}\angle 0^\circ) = 424.42\angle 29.02^\circ \ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 735.12 \ \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0018315)(424352\angle 0^\circ) + (0.8746)$$
$$(1604.357\angle 0^\circ) = 1604.04\angle 28.98^\circ \ \text{ A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 424.42\angle 29.02 \times 1604.04\angle -28.98^\circ \times 10^{-3}$$
$$= 2042.4 \text{ MW} + j1.4 \text{ Mvar}$$
$$= 2042.36\angle 0.04^\circ \quad \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{735.12}{0.8746} - 735}{735} \times 100 = 14.36\%$$

5.9. The transmission line of Problem 5.8 is energized with 765 kV at the sending end when the load at the receiving end is removed.

(a) Find the receiving end voltage.
(b) Determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 735 kV.
(a) The sending end voltage per phase is

$$V_S = \frac{765\angle 0^\circ}{\sqrt{3}} = 441.673\angle 0^\circ \quad \text{kV}$$

When line is open $I_R = 0$ and from (5.71) the no-load receiving end voltage is given by

$$V_{R(nl)} = \frac{V_S}{\cos \beta\ell} = \frac{441.673}{0.8746} = 505 \quad \text{kV}$$

The no-load receiving end line-to-line voltage is

$$V_{R(L-L)(nl)} = \sqrt{3}\, V_{R(nl)} = 874.68 \quad \text{kV}$$

(b) For $V_{RLL} = 735$ kV, the required inductor reactance given by (5.100) is

$$X_{Lsh} = \frac{\sin(29^\circ)}{\frac{765}{735} - \cos(29^\circ)}(264.7) = 772.13 \quad \Omega$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(KV_{Lrated})^2}{X_{Lsh}} = \frac{(735)^2}{772.13} = 699.65 \quad \text{Mvar}$$

5.10. The transmission line of Problem 5.8 is energized with 765 kV at the sending end when a three-phase short-circuit occurs at the receiving end. Determine the receiving end current and the sending end current.

For a solid short at the receiving end, $V_R = 0$ and from (5.71) and (5.72) we have

$$V_s = jZ_c \sin \beta\ell I_R$$

or

$$I_R = \frac{765000/\sqrt{3}}{j264.7\sin 29^\circ} = 3441.7\angle -90^\circ \quad \text{A}$$

$$I_S = \cos\beta\ell I_R = (\cos 29^\circ)(3441.7\angle -90^\circ) = 3010\angle -90^\circ \quad \text{A}$$

5.11. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.8. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. *Hint*: Use (5.93) and (5.94) to compute the power angle δ and the receiving end reactive power. Find the sending end quantities and voltage regulation for the compensated line.

(a) The equivalent line reactance for a lossless line is given by

$$X' = Z_c \ \sin\beta\ell = 264.7\sin(29^\circ) = 128.33 \ \ \Omega$$

The receiving end power is

$$S_{R(3\phi)} = 2000\angle\cos^{-1}(0.8) = 1600 + j1200 \quad \text{MVA}$$

For the above operating condition, the power angle δ is obtained from (5.93)

$$1600 = \frac{(765)(735)}{128.33} \ \sin\delta$$

which results in $\delta = 21.418^\circ$. Using the approximate relation given by (5.94), the net reactive power at the receiving end is

$$Q_{R(3\phi)} = \frac{(765)(735)}{128.33} \ \cos(21.418^\circ) - \frac{(735)^2}{128.33} \ \cos(29^\circ) = 397.05 \quad \text{Mvar}$$

Thus, the required capacitor Mvar is $S_C = j397.05 - j1200 = -j802.95$. The capacitive reactance is given by

$$X_C = \frac{|V_L|^2}{S_C^*} = \frac{(735)^2}{j802.95} = -j672.8 \ \ \Omega$$

or

$$C = \frac{10^6}{2\pi(60)(672.8)} = 3.9426 \ \ \mu\text{F}$$

The net receiving end complex power is

$$S_{R(3\phi)} = 1600 + j397.05 = 1648.53\angle 13.936^\circ \ \text{ MVA}$$

The receiving end current is

$$I_R = \frac{1600 - j397.05)10^3}{3(424.352\angle 0^\circ)} = 1294.94\angle{-13.9368^\circ} \ \text{ A}$$

We can check the sending end voltage

$$V_S = AV_R + BI_R = (0.8746)(424.352\angle 0^\circ) + (j128.33)$$
$$(1294.94 \times 10^{-3}\angle - 13.9368^\circ) = 441.67\angle 21.418^\circ \ \text{ KV}$$

or

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 765 \ \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0018315)(424352\angle 0^\circ) + (0.8746)$$
$$(1294.94\angle - 13.9368^\circ) = 1209.46\angle 24.65^\circ \ \text{ A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 441.67\angle 21.418 \times 1209.46\angle - 24.65^\circ \times 10^{-3} \\ &= 1600 \text{ MW} - j90.35 \text{ Mvar} \\ &= 1602.55\angle - 3.23^\circ \ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{765}{0.8746} - 735}{735} \times 100 = 19.00\%$$

5.12. Series capacitors are installed at the midpoint of the line of Problem 5.8, providing 40 percent compensation. Determine the sending end quantities and the voltage regulation when the line delivers 2000 MVA at 0.8 lagging power factor at 735 kV.

For 40 percent compensation, the series capacitor reactance per phase is

$$X_{ser} = 0.4 \times X' = 0.4(128.33) = 51.33 \ \ \Omega$$

or

$$C = \frac{10^6}{2\pi(60)(51.33)} = 51.67 \ \ \mu\text{F}$$

The new equivalent π circuit parameters are given by

$$Z' = j(X' - X_{ser}) = j(128.33 - 51.33) = j77 \;\; \Omega$$
$$Y' = = j\frac{2}{Z_c}\tan(\beta\ell/2) = j\frac{2}{264.7}\tan(29^\circ/2) = j0.001954 \;\text{ siemens}$$

The new B constant is $B = j77$ and the new A and C constants are given by

$$A = 1 + \frac{Z'Y'}{2} = 1 + \frac{(j77)(j0.001954)}{2} = 0.92476$$
$$C = Y'\left(1 + \frac{Z'Y'}{4}\right) = j0.001954\left(1 + \frac{(j77)(j0.001954)}{4}\right) = 0.0018805$$

The receiving end voltage per phase is

$$V_R = \frac{735}{\sqrt{3}} = 424.352 \quad \text{kV}$$

and the receiving end current is

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{2000\angle{-36.87^\circ}}{3 \times 424.35\angle 0^\circ} = 1.57102\angle{-36.87^\circ} \quad \text{kA}$$

Thus, the sending end voltage is

$$V_S = AV_R + BI_R = 0.92476 \times 424.352 + j77 \times 1.57102\angle{-36.87^\circ}$$
$$= 474.968\angle 11.756^\circ \;\; \text{kV}$$

and the line-to-line voltage magnitude is $|V_{S(L-L)}| = \sqrt{3}\,V_S = 822.67$ kV. The sending end current is

$$I_S = CV_R + DI_R = (j0.0018805)(424352\angle 0^\circ) + (0.92476)$$
$$(1571.02\angle - 36.87^\circ) = 1164.59\angle{-3.628^\circ} \;\; \text{A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I^*_S = 3 \times 474.968\angle 11.756 \times 1164.59\angle 3.628^\circ \times 10^{-3}$$
$$= 1600 \text{ MW} + j440.2 \text{ Mvar}$$
$$= 1659.4\angle 15.38^\circ \quad \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{822.67/0.92476 - 735}{735} \times 100 = 21.035\%$$

5.13. Series capacitors are installed at the midpoint of the line of Problem 5.8, providing 40 percent compensation. In addition, shunt capacitors are installed at the receiving end. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the series and shunt capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. Find the sending end quantities and voltage regulation for the compensated line.

The receiving end power is

$$S_{R(3\phi)} = 2000\angle\cos^{-1}(0.8) = 1600 + j1200 \quad \text{MVA}$$

With the series reactance $X' = 77\ \Omega$, and $\cos\beta\ell = A = 0.92476$, the power angle δ is obtained from (5.93)

$$1600 = \frac{(765)(735)}{77}\ \sin\delta$$

which results in $\delta = 12.657°$. Using the approximate relation given by (5.94), the net reactive power at the receiving end is

$$Q_{R(3\phi)} = \frac{(765)(735)}{77}\ \cos(12.657°) - \frac{(735)^2}{77}\ 0.92476 = 636.75 \quad \text{Mvar}$$

Thus, the required shunt capacitor Mvar is $S_C = j636.75 - j1200 = -j563.25$. The shunt capacitive reactance is given by

$$X_C = \frac{|V_L|^2}{S_C^*} = \frac{(735)^2}{j563.25} = -j959.12\ \ \Omega$$

or

$$C = \frac{10^6}{2\pi(60)(959.12)} = 2.765\ \ \mu\text{F}$$

The net receiving end complex power is

$$S_{R(3\phi)} = 1600 + j636.75 = 1722.05\angle 21.7°\ \ \text{MVA}$$

The receiving end current is

$$I_R = \frac{(1600 - j636.75)10^3}{3(424.352\angle 0°)} = 1352.69\angle{-21.7°}\ \ \text{A}$$

We can check the sending end voltage

$$V_S = AV_R + BI_R = (0.92476)(424.352\angle 0°) + (j77)$$
$$(1352.69 \times 10^{-3}\angle - 21.7°) = 441.67\angle 12.657°\ \ \text{kV}$$

or

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 765 \quad \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0018805)(424352\angle 0^\circ) + (0.92476)$$
$$(1352.69\angle -21.7^\circ) = 1209.7\angle 16.1^\circ \quad \text{A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 441.67\angle 12.657 \times 1209.7\angle -16.1^\circ \times 10^{-3}$$
$$= 1600 \text{ MW} - j96.3 \text{ Mvar}$$
$$= 1602.9\angle -3.44^\circ \quad \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{765}{0.92476} - 735}{735} \times 100 = 12.55\%$$

5.14. The transmission line of Problem 5.8 has a per phase resistance of $0.011\ \Omega$ per km. Using the **lineperf** program, perform the following analysis and present a summary of the calculation along with your conclusions and recommendations.

(a) Determine the sending end quantities for the specified receiving end quantities of $735\angle 0^\circ$ kV, 1600 MW, 1200 Mvar.
(b) Determine the receiving end quantities for the specified sending end quantities of $765\angle 0^\circ$ kV, 1920 MW, 600 Mvar.
(c) Determine the sending end quantities for a load impedance of $282.38 + j0\ \Omega$ at 735 kV.
(d) Find the receiving end voltage when the line is terminated in an open circuit and is energized with 765 kV at the sending end. Also, determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 765 kV. Obtain the voltage profile for the uncompensated and the compensated line.
(e) Find the receiving end and the sending end current when the line is terminated in a three-phase short circuit.
(f) For the line loading of part (a), determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV. Obtain the line performance of the compensated line.
(g) Determine the line performance when the line is compensated by series capacitor for 40 percent compensation with the load condition in part (a) at 735 kV.

(h) The line has 40 percent series capacitor compensation and supplies the load in part (a). Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV at the sending end.
(i) Obtain the receiving end circle diagram.
(j) Obtain the line voltage profile for a sending end voltage of 765 kV.
(k) Obtain the line loadability curves when the sending end voltage is 765 kV, and the receiving end voltage is 735 kV. The current-carrying capacity of the line is 5000 A per phase.

Use **lineperf** to perform all the above analyses.

5.15. The ABCD constants of a lossless three-phase, 500-kV transmission line are

$$A = D = 0.86 + j0$$
$$B = 0 + j130.2$$
$$C = j0.002$$

(a) Obtain the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

To improve the line performance, series capacitors are installed at both ends in each phase of the transmission line. As a result of this, the compensated ABCD constants become

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix}$$

where X_c is the total reactance of the series capacitor. If $X_c = 100\ \Omega$
(b) Determine the compensated ABCD constants.
(c) Determine the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

(a) The receiving end voltage per phase is

$$V_R = \frac{500\angle 0^\circ}{\sqrt{3}} = 288.675\angle 0^\circ \quad \text{kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 1000\angle \cos^{-1} 0.8 = 1000\angle 36.87^\circ = 800 + j600 \ \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{1000000\angle - 36.87^\circ}{3 \times 288.675\angle 0^\circ} = 1154.7\angle - 36.87^\circ \quad \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = (0.86)(288.675\angle 0^\circ) + (j130.2)$$
$$(1154.7 \times 10^{-3}\angle -36.87^\circ) = 359.2\angle 19.5626^\circ \ \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 622.153 \ \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.002)(288675\angle 0^\circ) + (0.86)$$
$$(1154.7\angle -36.87^\circ) = 794.647\angle -1.3322^\circ \ \text{A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 359.2\angle 19.5626 \times 794.647\angle 1.3322^\circ \times 10^{-3} \\ &= 800 \text{ MW} + j228.253 \text{ Mvar} \\ &= 831.925\angle 15.924^\circ \ \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{622.153}{0.86} - 500}{500} \times 100 = 44.687\%$$

(b) The compensated $ABCD$ constants are

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}j100 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.86 & j139.2 \\ j0.002 & 0.86 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}j100 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.96 & j39.2 \\ j0.002 & 0.96 \end{bmatrix}$$

(c) Repeating the analysis for the new $ABCD$ constants result in
The sending end voltage is

$$V_S = AV_R + BI_R = (0.96)(288.675\angle 0^\circ) + (j39.2)$$
$$(1154.7 \times 10^{-3}\angle -36.87^\circ) = 306.434\angle 6.7865^\circ \ \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 530.759 \ \text{kV}$$

$$I_S = CV_R + DI_R = (j0.002)(288675\angle 0^\circ) + (0.96)$$
$$(1154.7\angle -36.87^\circ) = 891.142\angle -5.6515^\circ \ \text{A}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 306.434\angle 6.7865 \times 891.142\angle 5.6515^\circ \times 10^{-3} \\ &= 800 \text{ MW} + j176.448 \text{ Mvar} \\ &= 819.227\angle 12.438^\circ \quad \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{530.759}{0.96} - 500}{500} \times 100 = 10.5748\%$$

We can use **lineperf** and option 4 to obtain the above solutions.

5.16. A three-phase 420-kV, 60-HZ transmission line is 463 km long and may be assumed lossless. The line is energized with 420 kV at the sending end. When the load at the receiving end is removed, the voltage at the receiving end is 700 kV, and the per phase sending end current is $646.6\angle 90^\circ$ A.

(a) Find the phase constant β in radians per Km and the surge impedance Z_c in Ω.
(b) Ideal reactors are to be installed at the receiving end to keep $|V_S| = |V_R| = 420$ kV when load is removed. Determine the reactance per phase and the required three-phase Mvar.

(a) The sending end and receiving end voltages per phase are

$$\begin{aligned} V_S &= \frac{420}{\sqrt{3}} = 242.487 \text{ kV} \\ V_{Rnl} &= \frac{700}{\sqrt{3}} = 404.145 \text{ kV} \end{aligned}$$

With load removed $I_R = 0$, from (5.71) we have

$$242.487 = (\cos \beta\ell)(404.145)$$

or

$$\beta\ell = 53.13^\circ = 0.927295 \text{ Radian}$$

and from (5.72), we have

$$j646.6 = j\frac{1}{Z_c}(\sin 53.13^\circ)(404.145)10^3$$

or

$$Z_c = 500 \ \Omega$$

(b) For $V_S = V_R$, the required inductor reactance given by (5.100) is

$$X_{Lsh} = \frac{\sin(53.13°)}{1 - \cos(53.13°)}(500) = 1000 \quad \Omega$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(KV_{Lrated})^2}{X_{Lsh}} = \frac{(420)^2}{1000} = 176.4 \quad \text{Mvar}$$

5.17. A three-phase power of 3600 MW is to be transmitted via four identical 60-Hz transmission lines for a distance of 300 Km. From a preliminary line design, the line phase constant and surge impedance are given by $\beta = 9.46 \times 10^{-4}$ radian/Km and $Z_c = 343\ \Omega$, respectively.
Based on the practical line loadability criteria determine the suitable nominal voltage level in kV for each transmission line. Assume $V_S = 1.0$ pu, $V_R = 0.9$ pu, and the power angle $\delta = 36.87°$.

$$\beta\ell = (9.46 \times 10^{-4})(300)(\frac{180}{\pi}) = 16.26°$$

The real power per transmission circuit is

$$P = \frac{3600}{4} = 900 \text{ MW}$$

From the practical line loadability given by(5.97), we have

$$900 = \frac{(1.0)(0.9)(SIL)}{\sin(16.26°)} \ \sin(36.87°)$$

Thus

$$SIL = 466.66 \quad \text{MW}$$

From (5.78)

$$KV_L = \sqrt{(Z_c)(SIL)} = \sqrt{(343)(466.66)} = 400 \quad \text{kV}$$

5.18. Power system studies on an existing system have indicated that 2400 MW are to be transmitted for a distance of 400 Km. The voltage levels being considered include 345 kV, 500 kV, and 765 kV. For a preliminary design based on the practical line loadability, you may assume the following surge impedances

345 kV $\quad Z_C = 320\ \Omega$
500 kV $\quad Z_C = 290\ \Omega$
765 kV $\quad Z_C = 265\ \Omega$

The line wavelength may be assumed to be 5000 Km. The practical line loadability may be based on a load angle δ of $35°$. Assume $|V_S| = 1.0$ pu and $|V_R| = 0.9$ pu. Determine the number of three-phase transmission circuits required for each voltage level. Each transmission tower may have up to two circuits. To limit the corona loss, all 500-kV lines must have at least two conductors per phase, and all 765-kV lines must have at least four conductors per phase. The bundle spacing is 45 cm. The conductor size should be such that the line would be capable of carrying current corresponding to at least 5000 MVA. Use **acsr** command in *MATLAB* to find a suitable conductor size. Following are the minimum recommended spacings between adjacent phase conductors at various voltage levels.

Voltage level, kV	Spacing meter
345	7.0
500	9.0
765	12.5

(a) Select a suitable voltage level, and conductor size, and tower structure. Use **lineperf** program and option 1 to obtain the voltage regulation and transmission efficiency based on a receiving end power of 3000 MVA at 0.8 power factor lagging at the selected rated voltage. Modify your design and select a conductor size for a line efficiency of at least 94 percent for the above specified load.
(b) Obtain the line performance including options 4–8 of the **lineperf** program for your final selection. Summarize the line characteristics and the required line compensation.

For each voltage level SIL is computed from (5.78). From the practical line loadability equation given by (5.97), the real power per circuit is computed, and number of circuits is established and tabulated in the following table.

Voltage KV	Surge Imp., Ω	SIL MW	Power/ circuit	No.of circuit
345	320	371.95	398.56	$\frac{2400}{398.56} = 6$
500	290	862.07	923.74	$\frac{2400}{923.74} = 3$
765	265	2208.39	2366.39	$\frac{2400}{2366.39} = 1$

For the 345 kV level we require six three-phase transmission circuit, and for the 500 kV line we need three circuits. Considering the cost of the transmission towers, right of ways and associated equipment, it can be concluded that one circuit at 765 kV is the most economical and practical choice.

Using the 765 kV for transmission line voltage, the per phase current corre-

sponding to 5000 MVA maximum is

$$I = \frac{5000 \times 10^3}{\sqrt{3}\,765} = 3773.5 \text{ A}$$

Since we have four conductor per phase, then current per conductor is

$$I_{cond} = \frac{3773.5}{4} = 943.38 \text{ A}$$

From the **acsr** file we select the CRANE conductor with a current-carrying capacity of 950 A. Typing acsr at the *MATLAB* prompt and selecting 'crane', result in

```
Enter ACSR code name within single quotes -> 'crane'
Al Area Strand  Diameter GMR  Resistance Ohm/km  Ampacity
 cmil   Al/St     cm      cm   60Hz 25C  60Hz 50C  Ampere
874500  54/7    2.9108   1.1765  0.06712   0.07632  950
```

Using **lineperf** result in

```
                    TRANSMISSION LINE MODEL
Type of parameters for input              Select

Parameters per unit length
r(ohm), g(seimens) L(mH) & C (micro F)       1

Complex z and y per unit length
r+j*x (ohm/length), g+j*b (seimens/length)   2

Nominal pi or Eq. pi model                   3

A, B, C, D constants                         4

Conductor configuration and dimension        5

To quit                                      0

Select number of menu --> 5
Enter spacing unit within quotes 'm' or 'ft'-> 'm'
Enter row vector [D12, D23, D13] = [12.5  12.5  25]
Cond. size, bundle spacing unit: Enter 'cm' or 'in'-> 'cm'
Conductor diameter in cm = 2.9108
Geometric Mean Radius in cm = 1.1765
No. of bundled cond. (enter 1 for single cond.) = 4
Bundle spacing in cm = 45
```

```
GMD  = 15.74901 m
GMRL =  0.19733 m   GMRC =  0.20811 m
L = 0.875934 mH/km     C = 0.0128525 micro F/km
Enter Line length = 400
Enter Frequency in Hz = 60
Enter line resistance/phase in ohms per unit length = 0.06712/4
Enter line conductance/phase in siemens per unit length = 0
Is the line model Short? Enter 'Y' or 'N' within quotes --> 'n'

Equivalent pi model
 --------------------
Z' = 6.15014 + j 126.538 ohms
Y' = 2.21291e-006 + j 0.00198054 siemens
Zc  = 261.145 + j -6.63073 ohms
alpha l = 0.0128511 neper beta l = 0.506128 radian = 28.999

        0.8747      + j 0.0062303    6.1501 + j 126.54
ABCD =
       -4.0954e-006 + j 0.0018565    0.8747 + j 0.0062303

    TRANSMISSION LINE PERFORMANCE
     ----------Analysis----------             Select
 To calculate sending end quantities
 for specified receiving end MW, Mvar           1

 To calculate receiving end quantities
 for specified sending end MW, Mvar             2

 To calculate sending end quantities
 when load impedance is specified               3

 Open-end line & inductive compensation         4

 Short-circuited line                           5

 Capacitive compensation                        6

 Receiving end circle diagram                   7

 Loadability curve and voltage profile          8

 To quit                                        0
 Select number of menu -->
```

Selecting options 1 through 8 result in

```
Line performance for specified receiving end quantities
-------------------------------------------------------
Vr = 765 kV (L-L)  at 0
Pr = 2400 MW   Qr = 1800 Mvar
Ir = 2264.12 A  at -36.8699  PFr = 0.8 lagging
Vs = 1059.49 kV (L-L)  at 21.4402
Is = 1630.56 A  at -12.6476  PFs = 0.82818 lagging
Ps = 2478.108   MW   Qs = 1677.037   Mvar
PL = 78.108     MW   QL = -122.963   Mvar
Percent Voltage Regulation   = 58.3312
Transmission line efficiency = 96.8481

Open line and shunt reactor compensation
----------------------------------------
Vs = 765 kV (L-L)  at 0
Vr = 874.563 kV (L-L)  at -0.00712268
Is = 937.387 A  at 89.7183  PFs = 0.00491663 leading

Desired no load receiving end voltage = 765 kV
Shunt reactor reactance = 1009.82 ohm
Shunt reactor rating = 579.531 Mvar
Hit return to continue

Line short-circuited at the receiving end
-----------------------------------------
Vs = 765 kV (L-L)  at 0
Ir = 3486.33 A  at -87.2174
Is = 3049.57 A  at -86.8093

Shunt capacitive compensation
-----------------------------
Vs = 765 kV (L-L)  at 31.8442
Vr = 765 kV (L-L)  at 0
Pload = 2400 MW   Qload = 1800 Mvar
Load current = 2264.12 A  at -36.8699  PFl = 0.8 lagging
Required shunt capcitor:287.826ohm,9.21593microF,2033.26 Mvar
Shunt capacitor current = 1534.51 A  at 90
Pr = 2400.000   MW   Qr = -233.261   Mvar
Ir = 1819.83 A  at 5.55126  PFr = 0.99531 leading
Is = 1863.22 A  at 31.9226  PFs = 0.999999 leading
Ps = 2468.802   MW   Qs = -3.378     Mvar
PL = 68.802     MW   QL = 229.882    Mvar
Percent Voltage Regulation   = 14.322
```

```
Transmission line efficiency = 97.2131

Series capacitor compensation
-----------------------------
Vr = 765 kV (L-L)  at 0
Pr = 2400 MW   Qr = 1800 Mvar
Required series capacitor:50.6151ohm,52.4069microF, 209.09Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Ir = 2264.12 A  at -36.8699  PFr = 0.8 lagging
Vs = 933.799 kV (L-L)  at 14.1603
Is = 1729.43 A  at -13.485  PFs = 0.885837 lagging
Ps = 2477.831   MW   Qs = 1297.875   Mvar
PL = 77.831     MW   QL = -502.125   Mvar
Percent Voltage Regulation   = 31.9848
Transmission line efficiency = 96.8589

Series and shunt capacitor compensation
---------------------------------------
Vs = 765 kV (L-L)  at 18.5228
Vr = 765 kV (L-L)  at 0
Pload = 2400 MW   Qload = 1800 Mvar
Load current = 2264.12 A  at -36.8699  PFl = 0.8 lagging
Required shunt capcitor: 322.574ohm, 8.22319microF,1814.24Mvar
Shunt capacitor current = 1369.22 A  at 90
Required series capacitor:50.6151ohm,52.4069microF,176.311Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Pr = 2400 MW   Qr = -14.2373 Mvar
Ir = 1811.33 A  at 0.339886  PFr = 0.999982 leading
Is = 1882.74 A  at 27.2821  PFs = 0.988337 leading
Ps = 2465.565   MW   Qs = -379.900   Mvar
PL = 65.565     MW   QL = -365.662   Mvar
Percent Voltage Regulation   = 8.12641
Transmission line efficiency = 97.3408
```

To transmit 2400 MW for a distance of 400 Km, we select a horizontal tower at 765 kV with four ACSR crane conductor per phase. The recommended compensators are:

- A three-phase shunt reactor for the open-ended line with a maximum rating of 580 Mvar at 765 kV.
- A three-phase shunt capacitor with a maximum rating of 1815 Mvar at 765 kV.
- A three-phase series capacitor with a maximum rating of 176 Mvar, 765 kV.

CHAPTER 6 PROBLEMS

6.1. A power system network is shown in Figure 47. The generators at buses 1 and 2 are represented by their equivalent current sources with their reactances in per unit on a 100-MVA base. The lines are represented by π model where series reactances and shunt reactances are also expressed in per unit on a 100 MVA base. The loads at buses 3 and 4 are expressed in MW and Mvar.
(a) Assuming a voltage magnitude of 1.0 per unit at buses 3 and 4, convert the loads to per unit impedances. Convert network impedances to admittances and obtain the bus admittance matrix by inspection.
(b) Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. (See Example 6.1.)

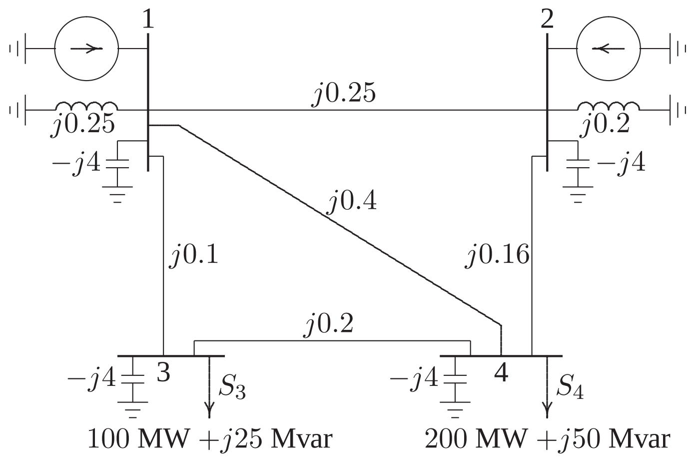


FIGURE 47
One-line diagram for Problem 6.1.

The load impedance in per unit is found from

$$Z = \frac{|V_{L-L}|^2}{S_L^*}\ \Omega \quad \& \quad Z_B = \frac{|V_B|^2}{S_B^*}\ \Omega \quad \text{or} \quad Z = \frac{|V_{pu}|^2}{S_{pu}^*}\ \text{ pu}$$

$$Z_3 = \frac{(1.0)^2}{1 - j0.25} = 0.9412 + j0.2353 \ \text{ pu}$$
$$Z_4 = \frac{(1.0)^2}{2 - j0.5} = 0.4706 + j0.11765 \ \text{ pu}$$

Converting all impedances to admittances results in the admittance diagram shown in Figure 48

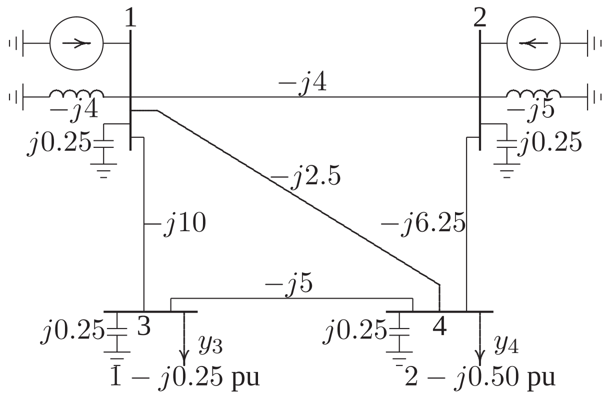


FIGURE 48
The admittance diagram for problem 6.1.

The self admittances are

$$Y_{11} = -j4 + j0.25 - j4 - j10 - j2.5 = -j20.25$$
$$Y_{22} = -j5 + j0.25 - j4 - j6.25 = -j15$$
$$Y_{33} = (1 - j0.25) + j0.25 - j10 - j5 = 1 - j15$$
$$Y_{44} = (2 - j0.5) + j0.25 - j2.5 - j6.25 - j5 = 2 - j14$$

Therefore, the bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} -j20.25 & j4 & j10 & j2.5 \\ j4 & -j15 & 0 & j6.25 \\ j10 & 0 & 1 - j15 & j5 \\ j2.5 & j6.25 & j5 & 2 - j14 \end{bmatrix}$$

From the impedance diagram the following data is constructed for use with the function **Y = ybus(Z)**

```
z = [0   1    0        0.25
     0   1    0       -4.0
     0   2    0        0.2
     0   2    0       -4.0
     0   3    0       -4.0
     0   3    0.9412   0.2353
     0   4    0       -4.0
     0   4    0.4706   0.1176
     1   2    0        0.25
     1   3    0        0.10
     1   4    0        0.40
     2   4    0        0.16
     3   4    0        0.20];
   Y=ybus(z)
```

The result is

```
Y =
     0 -20.25i     0 + 4.00i     0 +10.00i     0 + 2.50i
     0 + 4.00i     0 -15.00i     0             0 + 6.25i
     0 +10.00i     0             1 -15.00i     0 + 5.00i
     0 + 2.50i     0 + 6.25i     0 + 5.00i     2 -14.00i
```

6.2. A power system network is shown in Figure 49.

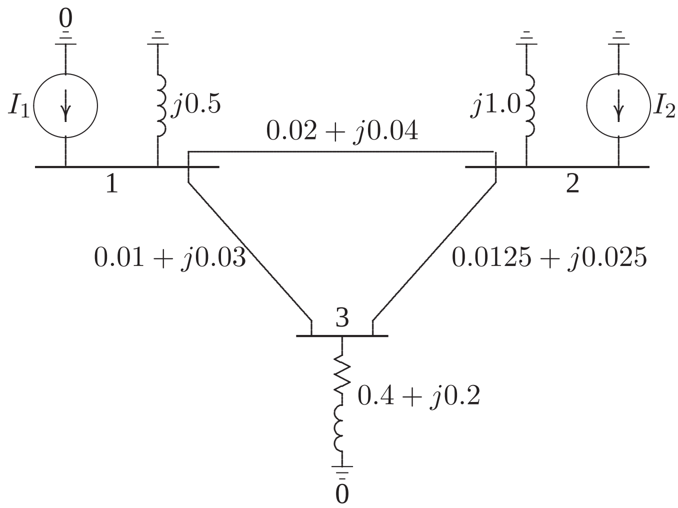


FIGURE 49
One-line diagram for Problem 6.2.

The values marked are impedances in per unit on a base of 100 MVA. The currents entering buses 1 and 2 are

$$I_1 = 1.38 - j2.72 \ \ \text{pu}$$
$$I_2 = 0.69 - j1.36 \ \ \text{pu}$$

(a) Determine the bus admittance matrix by inspection.
(b) Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. (See Example 6.1.) Write the necessary *MATLAB* commands to obtain the bus voltages.
Converting all impedances to admittances results in the admittance diagram shown in Figure 50

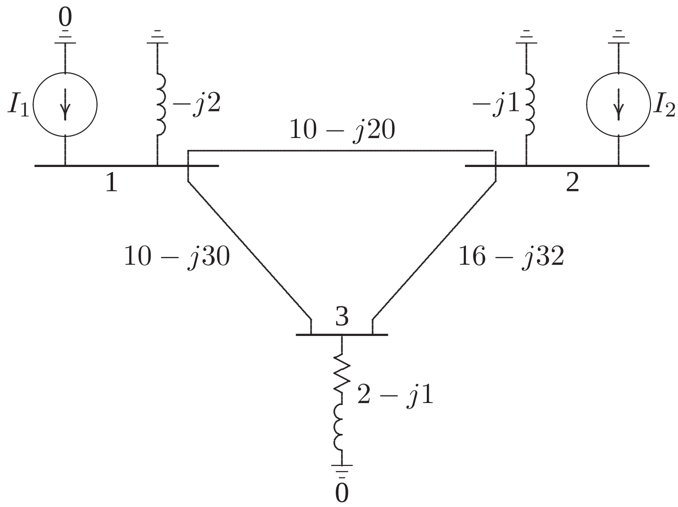


FIGURE 50
The admittance diagram for problem 6.2.

(a) The self admittances are

$$Y_{11} = -j2 + (10 - j20) + (10 - j30) = 20 - j52$$
$$Y_{22} = -j1 + (10 - j20) + (16 - j32) = 26 - j53$$
$$Y_{33} = (2 - j1) + (10 - j30) + (16 - j32) = 28 - j63$$

Therefore, the bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} 20 - j52 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j53 & -16 + j32 \\ -10 + j30 & -16 + j32 & 28 - j63 \end{bmatrix}$$

(b) To obtain the bus admittance matrix using **Y = ybus(Z)**, and the bus voltages, we use the following commands

```
z = [0   1    0.0     0.5
     0   2    0.0     1.0
     0   3    0.4     0.2
     1   2    0.02    0.04
     1   3    0.01    0.03
     2   3    0.0125  0.025];
Y=ybus(z)
I=[1.38-j*2.72;  0.69-j*1.36;  0];
V=Y\I;
Vm=abs(V)
phase = 180/pi*angle(V)
```

The result is

```
Y =
    20.0000-52.000i  -10.0000+20.000i  -10.0000+30.000i
   -10.0000+20.000i   26.0000-53.000i  -16.0000+32.000i
   -10.0000+30.000i  -16.0000+32.000i   28.0000-63.000i
Vm =
     1.0293
     1.0217
     1.0001
phase =
         1.4596
         0.9905
        -0.0150
```

6.3. Use Gauss-Seidel method to find the solution of the following equations

$$x_1 + x_1 x_2 = 10$$
$$x_1 + x_2 = 6$$

with the following initial estimates
(a) $x_1^{(0)} = 1$ and $x_2^{(0)} = 1$
(b) $x_1^{(0)} = 1$ and $x_2^{(0)} = 2$
Continue the iterations until $|\Delta x_1^{(k)}|$ and $|\Delta x_2^{(k)}|$ are less than 0.001.

Solving for x_1, and x_2 from the first and second equation respectively, results in

$$x_1 = \frac{10}{1 + x_2}$$
$$x_2 = 6 - x_1$$

(a) With initial estimates $x_1^{(0)} = 1$ and $x_2^{(0)} = 1$, the iterative sequence becomes

$$x_1^{(1)} = \frac{10}{1+1} = 5$$
$$x_2^{(1)} = 6 - 5 = 1$$
$$x_1^{(2)} = \frac{10}{1+1} = 5$$
$$x_2^{(2)} = 6 - 5 = 5$$

(b) With initial estimates $x_1^{(0)} = 1$ and $x_2^{(0)} = 2$, the iterative sequence becomes

$$x_1^{(1)} = \frac{10}{1+2} = 3.3333$$
$$x_2^{(1)} = 6 - 3.3333 = 2.6666$$
$$x_1^{(2)} = \frac{10}{1+2.6666} = 2.7272$$
$$x_2^{(2)} = 6 - 2.7272 = 3.2727$$
$$x_1^{(3)} = \frac{10}{1+3.2727} = 2.3404$$
$$x_2^{(3)} = 6 - 2.3404 = 3.6596$$
$$x_1^{(4)} = \frac{10}{1+3.6596} = 2.1461$$
$$x_2^{(4)} = 6 - 2.1461 = 3.8539$$
$$x_1^{(5)} = \frac{10}{1+3.8539} = 2.0602$$
$$x_2^{(5)} = 6 - 2.0602 = 3.9398$$
$$x_1^{(6)} = \frac{10}{1+3.9398} = 2.0244$$
$$x_2^{(6)} = 6 - 2.0244 = 3.9756$$
$$x_1^{(7)} = \frac{10}{1+3.9756} = 2.0098$$
$$x_2^{(7)} = 6 - 2.00098 = 3.9902$$
$$x_1^{(8)} = \frac{10}{1+3.9902} = 2.0039$$
$$x_2^{(8)} = 6 - 2.0039 = 3.9961$$
$$x_1^{(9)} = \frac{10}{1+3.9961} = 2.0016$$

$$x_2^{(9)} = 6 - 2.0244 = 3.9984$$
$$x_1^{(10)} = \frac{10}{1 + 3.9984} = 2.0006$$
$$x_2^{(10)} = 6 - 2.0006 = 3.9994$$

The above procedure may be placed in a *MATLAB* file as follows

```
x=input('Enter initial estimates [x1; x2] -> ');
dx=[1;1]    % Errors in x1, x2 are set to a high value
xc=x;
while max(abs(dx)) >=.001
xc(1)=10/(1+x(2));
xc(2)=6-xc(1);
dx=x-xc
x=xc
end
disp('Graphical display of of x1 and x2')
x2=0:.1:10;
x1=10*ones(1,length(x2))./(1+x2);
x3 =-x2+6;
plot(x2,x1,x2,x3),grid
```

The result is

```
Enter initial estimates [x1; x2] -> [1; 1]
x =
     5
     1
x =
     5
     1
Graphical display of of x1 and x2
```

Running **ch6p3** for the next initial estimates results in

```
Enter initial estimates [x1; x2] -> [1; 2]
x =
    3.3333
    2.6667
x =
    2.7273
    3.2727
x =
    2.3404
```

```
        3.6596
    x =
        2.1461
        3.8539
    x =
        2.0602
        3.9398
    x =
        2.0244
        3.9756
    x =
        2.0098
        3.9902
    x =
        2.0039
        3.9961
    x =
        2.0016
        3.9984
    x =
        2.0006
        3.9994
    Graphical display of of x1 and x2
```

6.4. A fourth-order polynomial equation is given by

$$x^4 - 21x^3 + 147x^2 - 379x + 252 = 0$$

(a) Use Newton-Raphson method and hand calculations to find one of the roots of the polynomial equation. Start with the initial estimate of $x^{(0)} = 0$ and continue until $|\Delta x^{(k)}| < 0.001$.
(b) Write a *MATLAB* program to find the roots of the above polynomial by Newton-Raphson method. The program should prompt the user to input the initial estimate. Run using the initial estimates of 0, 3, 6, 10.
(c) Check your answers using the *MATLAB* function **r = roots(A)**, where **A** is a row vector containing the polynomial coefficients in descending powers.

$$\frac{df(x)}{dx} = 4x^3 - 63x^2 + 294x - 379$$

(a) for $x^{(0)} = 0$, we have

$$\Delta c^{(0)} = c - f(x^{(0)}) = 0 - (252) = -252$$

$$\left(\frac{df}{dx}\right)^{(0)} = -379$$

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}} = \frac{-252}{-379} = 0.6649$$

Therefore, the result at the end of the first iteration is

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = 0 + 0.6649 = 0.6649$$

The subsequent iterations result in

$$\begin{aligned}\Delta c^{(1)} = c - f(x^{(1)}) &= 0 - [(0.6649)^4 - 21(0.6649)^3 + 147(0.6649)^2 \\ &-379(0.6649) + 252] = -59.0114\end{aligned}$$

$$\left(\frac{df}{dx}\right)^{(1)} = 4(0.6649)^3 - 63(0.6649)^2 + 294(0.6649) - 379 = -210.194$$

$$\Delta x^{(1)} = \frac{\Delta c^{(1)}}{\left(\frac{df}{dx}\right)^{(1)}} = \frac{-59.0114}{-210.194} = 0.28075$$

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = 0.6649 + 0.28075 = 0.9457$$

$$\begin{aligned}\Delta c^{(2)} = c - f(x^{(2)}) &= 0 - [(0.9457)^4 - 21(0.9457)^3 + 147(0.9457)^2 \\ &-379(0.9457) + 252] = -8.0942\end{aligned}$$

$$\left(\frac{df}{dx}\right)^{(2)} = 4(0.9457)^3 - 63(0.9457)^2 + 294(0.9457) - 379 = -153.9333$$

$$\Delta x^{(2)} = \frac{\Delta c^{(2)}}{\left(\frac{df}{dx}\right)^{(2)}} = \frac{-8.0942}{-153.9333} = 0.05258$$

$$x^{(3)} = x^{(2)} + \Delta x^{(2)} = 0.9457 + 0.05258 = 0.9982$$

$$\Delta c^{(3)} = c - f(x^{(3)}) = 0 - [(0.9982)^4 - 21(0.9982)^3 + 147(0.9982)^2$$
$$-379(0.9982) + 252] = -0.2541$$

$$\left(\frac{df}{dx}\right)^{(3)} = 4(0.9982)^3 - 63(0.9982)^2 + 294(0.9982) - 379 = -144.3174$$

$$\Delta x^{(3)} = \frac{\Delta c^{(3)}}{\left(\frac{df}{dx}\right)^{(3)}} = \frac{-0.2541}{-144.3174} = 0.0018$$

$$x^{(4)} = x^{(3)} + \Delta x^{(3)} = 0.9982 + 0.0018 = 1.0000$$

$$\Delta c^{(4)} = c - f(x^{(4)}) = 0 - [(1.0000)^4 - 21(1.0000)^3 + 147(1.0000)^2$$
$$-379(1.0000) + 252] = -0.0003$$

$$\left(\frac{df}{dx}\right)^{(4)} = 4(1.0000)^3 - 63(1.0000)^2 + 294(1.0000) - 379 = -144.0003$$

$$\Delta x^{(4)} = \frac{\Delta c^{(4)}}{\left(\frac{df}{dx}\right)^{(4)}} = \frac{-0.0003}{-144.0003} = 0.0000$$

$$x^{(5)} = x^{(4)} + \Delta x^{(4)} = 1.0000 + 0.0000 = 1.0000$$

The following commands show the procedure for the solution of the given equation by the Newton-Raphson method.

```
dx=1;              % Change in variable is set to a high value
x=input('Enter the initial estimate -> ');  % Initial estimate
iter = 0;                                     % Iteration counter
disp('iter   Dc       J        dx       x')%Heading for result
while abs(dx) >= 0.001 & iter < 100     % Test for convergence
iter = iter + 1;                              % No. of iterations
Dc=0 - (x^4-21*x^3+147*x^2-379*x+252);                % Residual
J = 4*x^3-63*x^2+ 294*x-379;                        % Derivative
dx= Dc/J;                                     %Change in variable
x=x+dx;                                     % Successive solution
fprintf('%g', iter), disp([Dc, J, dx, x])
```

The result is

```
Enter the initial estimate -> 0

iter   Dc        J         dx        x
1 -252.0000 -379.0000    0.6649    0.6649

2  -59.0114 -210.1938    0.2807    0.9457

3   -8.0942 -153.9333    0.0526    0.9982

4   -0.2541 -144.3174    0.0018    1.0000

5   -0.0003 -144.0003    0.0000    1.0000
```

Repeating for $x^{(0)} = 3$, we have

```
Enter the initial estimate -> 3
iter   Dc        J         dx        x
1   48.0000   44.0000    1.0909    4.0909

2   -4.0128   43.2427   -0.0928    3.9981

3    0.0850   45.0339    0.0019    4.0000

4    0.0000   45.0000    0.0000    4.0000
```

Repeating for $x^{(0)} = 6$, we have

```
Enter the initial estimate -> 6
iter   Dc        J         dx        x
1  -30.0000  -19.0000    1.5789    7.5789

2   19.3714  -28.1850   -0.6873    6.8917

3   -3.8917  -35.7586    0.1088    7.0005

4    0.0175  -36.0000   -0.0005    7.0000
```

Repeating for $x^{(0)} = 10$, we have

```
Enter the initial estimate -> 10
```

```
iter   Dc         J          dx         x
1 -162.0000  261.0000   -0.6207    9.3793

2  -40.6800  136.7617   -0.2975    9.0819

3   -6.9992   91.1091   -0.0768    9.0050

4   -0.4046   80.6659   -0.0050    9.0000

5   -0.0017   80.0027    0.0000    9.0000
```

The above initial estimates have converged to the values of 1, 4, 7, and 9. As we can see from the *MATLAB* **roots** function, these are the roots of the above fourth order polynomial.

```
A = [1 -21  147  -379  252];
r = roots(A)

r =
    9.0000
    7.0000
    4.0000
    1.0000
```

6.5. Use Newton-Raphson method and hand calculation to find the solution of the following equations:

$$x_1^2 - 2x_1 - x_2 = 3$$
$$x_1^2 + x_2^2 = 41$$

(a) Start with the initial estimates of $x_1^{(0)} = 2$, $x_2^{(0)} = 3$. Perform three iterations.
(b) Write a *MATLAB* program to find one of the solutions of the above equations by Newton-Raphson method. The program should prompt the user to input the initial estimates. Run the program with the above initial estimates.

Taking partial derivatives of the above equations results in the Jacobian matrix

$$J = \begin{bmatrix} 2x_1 - 2 & -1 \\ 2x_1 & 2x_2 \end{bmatrix}$$

(a) Starting with initial estimates $x_1^{(0)} = 2$, and $x_2^{(0)} = 3$, the analytical solution given by the Newton-Raphson method is

$$\Delta C^{(0)} = \begin{bmatrix} 3 - [(2)^2 - 2(2) - 3] \\ 41 - [(2)^2 + (3)^2] \end{bmatrix} = \begin{bmatrix} 6 \\ 28 \end{bmatrix}$$

$$J = \begin{bmatrix} 2(2) - 2 & -1 \\ 2(2) & 2(3) \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$$

$$\Delta X^{(0)} = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 28 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$X^{(0)} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

For the second iteration, we have

$$\Delta C^{(1)} = \begin{bmatrix} 3 - [(6)^2 - 2(6) - 5] \\ 41 - [(6)^2 + (5)^2] \end{bmatrix} = \begin{bmatrix} -16 \\ -20 \end{bmatrix}$$

$$J^{(1)} = \begin{bmatrix} 2(6) - 2 & -1 \\ 2(6) & 2(5) \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 12 & 10 \end{bmatrix}$$

$$\Delta X^{(1)} = \begin{bmatrix} 10 & -1 \\ 12 & 10 \end{bmatrix}^{-1} \begin{bmatrix} -16 \\ -20 \end{bmatrix} = \begin{bmatrix} -1.6071 \\ -0.0714 \end{bmatrix}$$

$$X^{(2)} = \begin{bmatrix} 6 \\ 5 \end{bmatrix} + \begin{bmatrix} -1.6071 \\ -0.0714 \end{bmatrix} = \begin{bmatrix} 4.3929 \\ 4.9286 \end{bmatrix}$$

and for the third iteration, we have

$$\Delta C^{(2)} = \begin{bmatrix} 3 - [(4.3929)^2 - 2(4.3929) - 4.9286] \\ 41 - [(4.3929)^2 + (4.9286)^2] \end{bmatrix} = \begin{bmatrix} -2.5829 \\ -2.5880 \end{bmatrix}$$

$$J^{(2)} = \begin{bmatrix} 2(4.3929) - 2 & -1 \\ 2(4.3929) & 2(4.9286) \end{bmatrix} = \begin{bmatrix} 6.7857 & -1.0000 \\ 8.7857 & 10.000 \end{bmatrix}$$

$$\Delta X^{(1)} = \begin{bmatrix} 6.7857 & -1.0000 \\ 8.7857 & 10.000 \end{bmatrix}^{-1} \begin{bmatrix} -2.5829 \\ -2.5880 \end{bmatrix} = \begin{bmatrix} -0.3706 \\ 0.0678 \end{bmatrix}$$

$$X^{(3)} = \begin{bmatrix} 4.3929 \\ 4.9286 \end{bmatrix} + \begin{bmatrix} -0.3706 \\ 0.0678 \end{bmatrix} = \begin{bmatrix} 4.0222 \\ 4.9964 \end{bmatrix}$$

The following *MATLAB* program is used to find one of the solutions of the above equations.

```
iter = 0;                                 % Iteration counter
x=input('Enter initial estimates, col. vector [x1; x2]-> ');
Dx = [1; 1];
C=[3; 41];
disp('Iter   DC       Jacobin matrix       Dx        x');
                                           % Heading for results
while max(abs(Dx)) >= .0001  & iter < 10 % Convergence test
iter=iter+1;                                 % Iteration counter
f = [x(1)^2-2*x(1)-x(2);  x(1)^2+x(2)^2];       % Functions
DC = C - f;                                        % Residuals
J = [2*x(1)-2       -1                        % Jacobian matrix
     2*x(1)       2*x(2)];
Dx=J\DC;                                   % Change in variables
x=x+Dx;                                   % Successive solutions
fprintf('%g', iter), disp([DC, J, Dx, x])         % Results
end
```

The result is

```
Enter initial estimates, col. vector [x1; x2] -> [2; 3]
Iter   DC      Jacobin matrix        Dx         x
1      6          2         -1          4         6
       28         4          6          2         5

2   -16.0000   10.0000   -1.0000   -1.6071    4.3929
    -20.0000   12.0000   10.0000   -0.0714    4.9286

3    -2.5829    6.7857   -1.0000   -0.3706    4.0222
     -2.5880    8.7857    9.8571    0.0678    4.9964

4    -0.1374    6.0444   -1.0000   -0.0221    4.0001
     -0.1420    8.0444    9.9928    0.0036    5.0000

5    -0.0005    6.0002   -1.0000   -0.0001    4.0000
     -0.0005    8.0002   10.0000    0.0000    5.0000
```

6.6. In the power system network shown in Figure 51, bus 1 is a slack bus with $V_1 = 1.0\angle 0^\circ$ per unit and bus 2 is a load bus with $S_2 = 280$ MW + $j60$ Mvar. The line impedance on a base of 100 MVA is $Z = 0.02 + j0.04$ per unit.

(a) Using Gauss-Seidel method, determine V_2 . Use an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and perform four iterations.

(b) If after several iterations voltage at bus 2 converges to $V_2 = 0.90 - j0.10$, determine S_1 and the real and reactive power loss in the line.

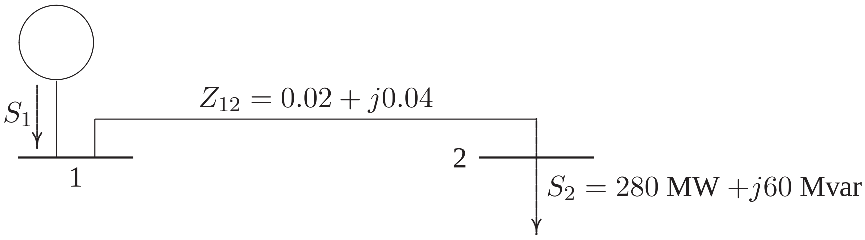


FIGURE 51
One-line diagram for Problem 6.6.

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j29$$

The per unit load at bus 2 is

$$S_2 = -\frac{280 + j60}{100} = -2.8 - j0.60$$

Starting with an initial estimate of $V_2^{(0)} = 1.0 + j0.0$, the voltage at bus 2 computed from (6.28) for three iterations are

$$V_2^{(1)} = \frac{\frac{-2.8+j0.60}{1.00000-j0.00000} + (10 - j20)(1)}{10 - j20} = 0.92000 - j0.10000$$
$$V_2^{(2)} = \frac{\frac{-2.8+j0.60}{0.92000+j0.10000} + (10 - j20)(1)}{10 - j20} = 0.90238 - j0.09808$$
$$V_2^{(3)} = \frac{\frac{-2.8+j0.60}{0.90238-j0.09808} + (10 - j20)(1)}{10 - j20} = 0.90050 - j0.10000$$

(b) Assuming voltage at bus 2 converges to $V_2 = 0.9 - j0.1$, the line flows are computed as follows

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1 + j0) - (0.9 - j0.10] = 3.0 - j1.0$$

$$I_{21} = -I_{12} = -3.0 + j1.0$$

$$\begin{aligned} S_{12} = V_1 I_{12}^* = (1.0 + j0.0)(3.0 + j1.0) &= 3 + j1 \text{ pu} \\ &= 300 \text{ MW} + j100 \text{ Mvar} \\ S_{21} = V_2 I_{21}^* = (0.9 - j0.1)(-3.0 - j1.0) &= -2.8 - j0.6 \text{ pu} \\ &= -280 \text{ MW} - j60 \text{ Mvar} \end{aligned}$$

The line loss is

$$S_{L12} = S_{12} + S_{21} = (300 + j100) + (-280 - j60) = 20 \text{ MW} + j40 \text{ Mvar}$$

The slack bus real and reactive power are $P_1 = 300$ MW, and $Q_1 = 100$ Mvar. The above calculations is placed in a *MATLAB* file, and solution is obtained by typing **ch6p6** at the *MATLAB prompt*.

6.7. Figure 6.6 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0\angle 0^\circ$ per unit. The scheduled loads on buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.
(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, determine V_2 and V_3. Perform two iterations.
(b) If after several iterations the bus voltages converge to

$$V_2 = 0.90 - j0.10 \ \text{ pu}$$
$$V_3 = 0.95 - j0.05 \ \text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.
(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.) Use a power accuracy of 0.00001 and an acceleration factor of 1.0.

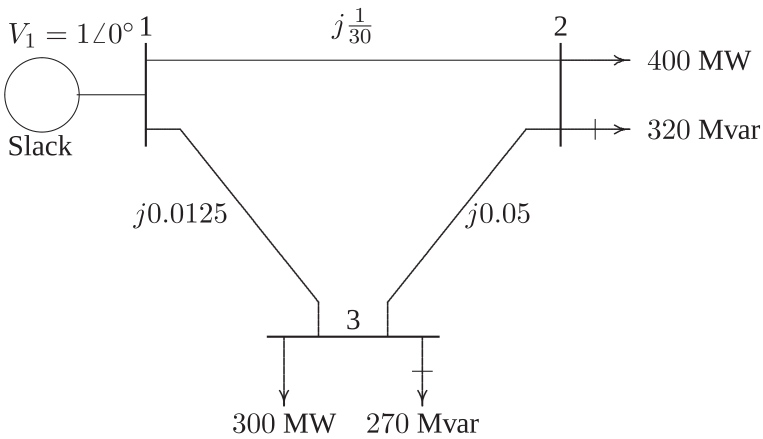


FIGURE 52
One-line diagram for Problem 6.7.

(a) Line impedances are converted to admittances

$$y_{12} = -j30$$
$$y_{13} = \frac{1}{j0.0125} = -j80$$
$$y_{23} = \frac{1}{j0.05} = -j20$$

At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(400 + j320)}{100} = -4.0 - j3.2 \text{ pu}$$
$$S_3^{sch} = -\frac{(300 + j270)}{100} = -3.0 - j2.7 \text{ pu}$$

For hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$V_2^{(1)} = \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$
$$= \frac{\frac{-4.0+j3.2}{1.0-j0} + (-j30)(1.0 + j0) + (-j20)(1.0 + j0)}{-j50}$$
$$= 0.936 - j0.08$$

and

$$V_3^{(1)} = \frac{\frac{S_3^{sch*}}{V_3^{(0)*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$
$$= \frac{\frac{-3.0+j2.7}{1-j0} + (-j80)(1.0 + j0) + (-j20)(0.936 - j0.08)}{-j100}$$
$$= 0.9602 - j0.046$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-4.0+j3.2}{0.936+j0.08} + (-j30)(1.0 + j0) + (-j20)(0.9602 - j0.046)}{-j50}$$
$$= 0.9089 - j0.0974$$

and

$$V_3^{(2)} = \frac{\frac{-3.0+j2.7}{0.9602+j0.046} + (-j80)(1.0 + j0) + (-j20)(0.9089 - j0.0974)}{(-j100)}$$
$$= 0.9522 - j0.0493$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} per unit in seven iterations as given below.

$$
\begin{aligned}
V_2^{(3)} &= 0.9020 - j0.0993 & V_3^{(3)} &= 0.9505 - j0.0498 \\
V_2^{(4)} &= 0.9004 - j0.0998 & V_3^{(4)} &= 0.9501 - j0.0500 \\
V_2^{(5)} &= 0.9001 - j0.1000 & V_3^{(5)} &= 0.9500 - j0.0500 \\
V_2^{(6)} &= 0.9000 - j0.1000 & V_3^{(6)} &= 0.9500 - j0.0500 \\
V_2^{(7)} &= 0.9000 - j0.1000 & V_3^{(7)} &= 0.9500 - j0.0500
\end{aligned}
$$

The final solution is

$$
\begin{aligned}
V_2 &= 0.90 - j0.10 = 0.905554\angle{-6.34^\circ} \quad \text{pu} \\
V_3 &= 0.95 - j0.05 = 0.9513\angle{-3.0128^\circ} \quad \text{pu}
\end{aligned}
$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$
\begin{aligned}
P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\
&= 1.0[1.0(-j30 - j80) - (-j30)(0.9 - j0.1) - \\
&\quad (-j80)(0.95 - j0.05)] \\
&= 7.0 - j7.0
\end{aligned}
$$

or the slack bus real and reactive powers are $P_1 = 7.0$ pu = 700 MW and $Q_1 = 7.0$ pu $=$ 700 Mvar.

To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$
\begin{aligned}
I_{12} &= y_{12}(V_1 - V_2) = (-j30)[(1.0 + j0) - (0.90 - j0.10)] = 3.0 - j3.0 \\
I_{21} &= -I_{12} = -3.0 + j3.0 \\
I_{13} &= y_{13}(V_1 - V_3) = (-j80)[(1.0 + j0) - (0.95 - j.05)] = 4.0 - j4.0 \\
I_{31} &= -I_{13} = -4.0 + j4.0 \\
I_{23} &= y_{23}(V_2 - V_3) = (-j20)[(0.90 - j0.10) - (0.95 - j.05)] = -1.0 + j1.0 \\
I_{32} &= -I_{23} = 1.0 - j1.0
\end{aligned}
$$

The line flows are

$$
\begin{aligned}
S_{12} = V_1 I_{12}^* &= (1.0 + j0.0)(3.0 + j3) = 3.0 + j3.0 \ \text{pu} \\
&= 300 \ \text{MW} + j300 \ \text{Mvar}
\end{aligned}
$$

$$
\begin{aligned}
S_{21} = V_2 I_{21}^* &= (0.90 - j0.10)(-3 - j3) = -3.0 - j2.4 \text{ pu} \\
&= -300 \text{ MW} - j240 \text{ Mvar} \\
S_{13} = V_1 I_{13}^* &= (1.0 + j0.0)(4.0 + j4.0) = 4.0 + j4.0 \text{ pu} \\
&= 400 \text{ MW} + j400 \text{ Mvar} \\
S_{31} = V_3 I_{31}^* &= (0.95 - j0.05)(-4.0 - j4.0) = -4.0 - j3.6 \text{ pu} \\
&= -400 \text{ MW} - j360 \text{ Mvar} \\
S_{23} = V_2 I_{23}^* &= (0.90 - j0.10)(-1.0 - j1.0) = -1.0 - j0.80 \text{ pu} \\
&= -100 \text{ MW} - j80 \text{ Mvar} \\
S_{32} = V_3 I_{32}^* &= (0.95 - j0.05)(1 + j1) = 1.0 + j0.9 \text{ pu} \\
&= 100 \text{ MW} + j90 \text{ Mvar}
\end{aligned}
$$

and the line losses are

$$
\begin{aligned}
S_{L\,12} &= S_{12} + S_{21} = 0.0 \text{ MW} + j60 \text{ Mvar} \\
S_{L\,13} &= S_{13} + S_{31} = 0.0 \text{ MW} + j40 \text{ Mvar} \\
S_{L\,23} &= S_{23} + S_{32} = 0.0 \text{ MW} + j10 \text{ Mvar}
\end{aligned}
$$

The power flow diagram is shown in Figure 6.7, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

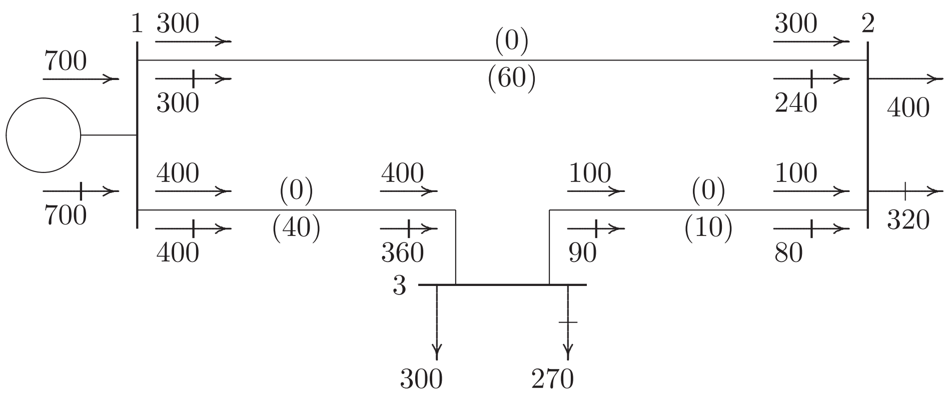


FIGURE 53
Power flow diagram of Problem 6.7 (powers in MW and Mvar).

(c) The power flow program **lfgauss** is used to obtain the solution, with the following statements:

```
clear
basemva = 100;  accuracy = 0.000001; accel = 1.1; maxiter = 100;
```

```
%      Problem   6.7(c)
%      Bus  Bus Voltage Angle -Load---   -Generator-- Injected
%       No code Mag. Degree MW   MVAR  MW  MVAR Qmin Qmax  Mvar
busdata=[1   1  1.0   0.0  0.0  0.0   0.0  0.0   0   0    0
         2   0  1.0   0.0  400  320   0.0  0.0   0   0    0
         3   0  1.0   0.0  300  270   0.0  0.0   0   0    0];

%                                  Line code
%        Bus bus   R     X    1/2 B   = 1 for lines
%         nl  nr pu    pu     pu    >1 or <1 tr. tap at bus nl
linedata=[1   2  0.0  1/30    0.0       1
          1   3  0.0  0.0125  0.0       1
          2   3  0.0  0.050   0.0       1];
disp('Problem 6.7(c)')
lfybus                          % form the bus admittance matrix
lfgauss               % Load flow solution by Gauss-Seidel method
busout             % Prints the power flow solution on the screen
lineflow         % Computes and displays the line flow and losses
```

The above statements are saved in the file **ch6p7c.m**. Run the program to obtain the solution.

6.8. Figure 54 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 3. The voltage at bus 1 is $V_1 = 1.025\angle 0°$ per unit. Voltage magnitude at bus 3 is fixed at 1.03 pu with a real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0+j0$ and $V_3^{(0)} = 1.03 + j0$ and keeping $|V_3| = 1.03$ pu, determine the phasor values of V_2 and V_3. Perform two iterations.
(b) If after several iterations the bus voltages converge to

$$V_2 = 1.001243\angle{-2.1°} = 1.000571 - j0.0366898 \;\text{ pu}$$
$$V_3 = 1.03\angle 1.36851° = 1.029706 + j.0246 \;\text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.
(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.)

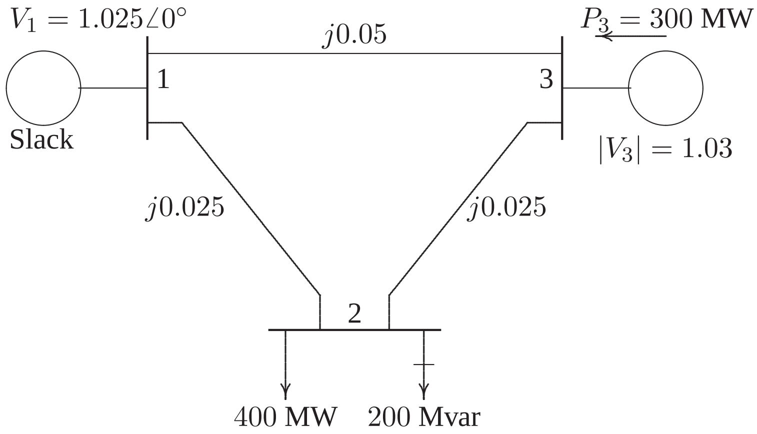


FIGURE 54
One-line diagram of Problem 6.8.

(a) Line impedances converted to admittances are $y_{12} = -j40$, $y_{13} = -j20$ and $y_{23} = -j40$. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j200)}{100} = -4.0 - j2.0 \text{ pu}$$
$$P_3^{sch} = \frac{300}{100} = 3.0 \text{ pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.03 + j0.0$, V_2 and V_3 are computed from (6.28).

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0+j2.0}{1.0-j0} + (-j40)(1.025 + j0) + (-j40)(1.03 + j0)}{(-j80)} \\ &= 1.0025 - j0.05 \end{aligned}$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$\begin{aligned} Q_3^{(1)} &= -\Im\{V_3^{(0)*}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\ &= -\Im\{(1.03 - j0)[(1.03 + j0)(-j60) - (-j20)(1.025 + j0) \\ &\quad -(-j40)(1.0025 - j0.05)]\} \\ &= 1.236 \end{aligned}$$

The value of $Q_3^{(1)}$ is used as Q_3^{sch} for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$\begin{aligned} V_{c3}^{(1)} &= \frac{\frac{S_3^{sch}*}{V_3^{(0)}*} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13}+y_{23}} \\ &= \frac{\frac{3.0-j1.236}{1.03-j0} + (-j20)(1.025+j0) + (-j40)(1.0025-j0.05)}{(-j60)} \\ &= 1.0300 + j0.0152 \end{aligned}$$

Since $|V_3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = 0.0152$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.03)^2 - (0.0152)^2} = 1.0299$$

Thus

$$V_3^{(1)} = 1.0299 + j0.0152$$

For the second iteration, we have

$$\begin{aligned} V_2^{(2)} &= \frac{\frac{S_2^{sch}*}{V_2^{(1)^*}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12}+y_{23}} \\ &= \frac{\frac{-4.0+j2.0}{1.0025+j.05} + (-j40)(1.025) + (-j40)(1.0299+j0.0152)}{(-j80)} \\ &= 1.0001 - j0.0409 \end{aligned}$$

$$\begin{aligned} Q_3^{(2)} &= -\Im\{V_3^{(1)^*}[V_3^{(1)}(y_{13}+y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\} \\ &= -\Im\{(1.0299-j0.0152)[(1.0299+j0.0152)(-j60) \\ &\quad -(-j20)(1.025+j0) - (-j40)(1.0001-j0.0409)]\} \\ &= 1.3671 \end{aligned}$$

$$\begin{aligned} V_{c3}^{(2)} &= \frac{\frac{S_3^{sch}*}{V_3^{(1)^*}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13}+y_{23}} \\ &= \frac{\frac{3.0-j1.3671}{1.0299-j0.0152} + (-j20)(1.025) + (-j40)(1.0001-j.0409)}{(-j60)} \\ &= 1.0298 + j0.0216 \end{aligned}$$

Since $|V_3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = 0.0216$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.03)^2 - (0.0216)^2} = 1.0298$$

or

$$V_3^{(2)} = 1.0298 + j0.0216$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} pu to

$$V_2 = 1.001243\angle{-2.1^\circ} = 1.000571 - j.0366898 \text{ pu}$$
$$S_3 = 3.0 + j1.3694 \text{ pu} = 300 \text{ MW} + j136.94 \text{ Mvar}$$
$$V_3 = 1.03\angle 1.36851^\circ \text{ pu} = 1.029706 + j0.0246$$

(b) Line flows and line losses are computed as in Problem 6.7, and the results expressed in MW and Mvar are

$$S_{12} = 150.43 + j100.16 \quad S_{21} = -150.43 - j92.39 \quad S_{L\,12} = 0.0 + j7.77$$
$$S_{13} = -50.43 - j9.65 \quad S_{31} = 50.43 + j10.90 \quad S_{L\,13} = 0.0 + j1.25$$
$$S_{23} = -249.58 - j107.61 \quad S_{32} = 249.58 + j126.03 \quad S_{L\,23} = 0.0 + j18.42$$

The slack bus real and reactive powers are

$$S_1 = S_{12} + S_{13} = (150.43 + j100.16) + (-50.43 - j9.65)$$
$$= 100 \text{ MW} + j90.51 \text{ Mvar}$$

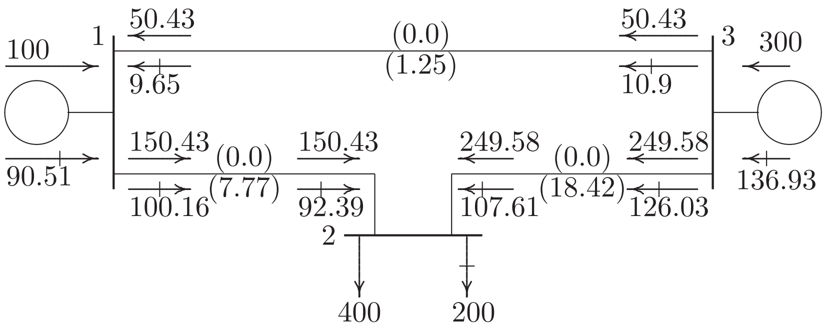


FIGURE 55
Power flow diagram of Problem 6.8 (powers in MW and Mvar).

The power flow diagram is shown in Figure 55, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parenthesis are the real and reactive losses in the line.

(c) The power flow program **lfgauss** is used to obtain the solution, with the following statements:

```
clear
basemva = 100;  accuracy = 0.000001; accel = 1.1; maxiter = 100;

%       Problem   6.8(c)
%       Bus  Bus Voltage Angle -Load---   -Generator-- Injected
%        No code Mag. Degree MW  MVAR  MW  MVAR Qmin Qmax  Mvar
busdata=[1   1  1.025 0.0  0.0  0.0   0.0  0.0   0   0    0
         2   0  1.0   0.0  400  200   0.0  0.0   0   0    0
         3   2  1.03  0.0  0.0  0.0   300  0.0   0 160    0];

%                                   Line code
%         Bus bus   R    X   1/2 B  = 1 for lines
%          nl  nr pu    pu      pu  >1 or <1 tr. tap at bus nl
linedata=[1   2  0.0  0.025   0.0       1
          1   3  0.0   0.05   0.0       1
          2   3  0.0  0.025   0.0       1];
disp('Problem 6.8(c)')
lfybus                       % form the bus admittance matrix
lfgauss            % Load flow solution by Gauss-Seidel method
busout          % Prints the power flow solution on the screen
lineflow      % Computes and displays the line flow and losses
```

The above statements are saved in the file **ch6p8c.m**. Run the program to obtain the solution.

6.9. The one-line diagram of a four-bus power system is as shown in Figure 56. Reactances are given in per unit on a common MVA base. Transformers T_1 and T_2 have tap settings of 0.8:1, and 1.25:1 respectively. Obtain the bus admittance matrix.

From Figure 6.15 in the text, the sending end transformer π model is

$$y_{12} = \frac{y_t}{a} = \frac{-j80}{0.8} = -j100$$
$$y_{10} = \left(\frac{1-a}{a^2}\right) y_t = \frac{1-0.8}{0.64}(-j80) = -j25$$
$$y_{30} = \left(\frac{a-1}{a}\right) y_t = \frac{0.8-1}{0.8}(-j80) = j20$$

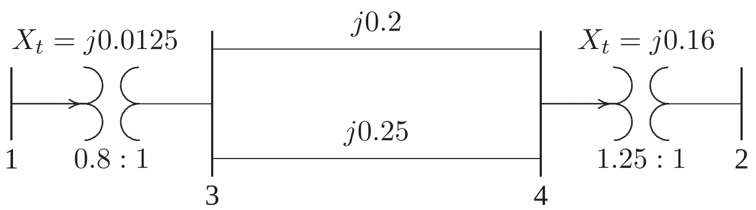


FIGURE 56
One-line diagram for Problem 6.9.

and the receiving end transformer π model is

$$y_{42} = \frac{y_t}{a} = \frac{-j6.25}{1.25} = -j5$$
$$y_{40} = \left(\frac{1-a}{a^2}\right) y_t = \frac{1-1.25}{1.5625}(-j6.25) = j1$$
$$y_{20} = \left(\frac{a-1}{a}\right) y_t = \frac{1.25-1}{1.25}(-j6.25) = -j1.25$$

All line impedances are converted to admittances, and the admittance diagram is constructed as shown in Figure 57.

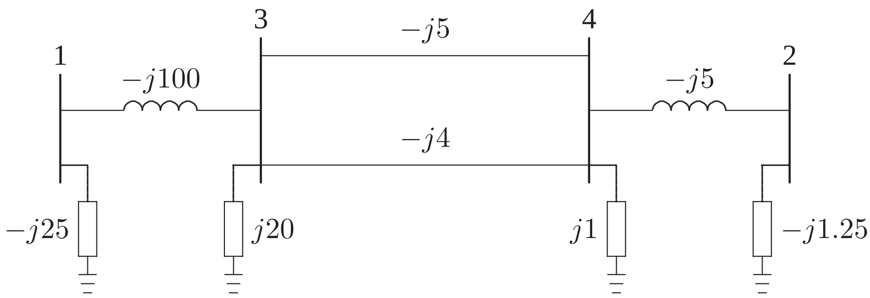


FIGURE 57
Admittance diagram for the system of Problem 6.9.

The bus admittance matrix for the network of Figure 57 obtained by inspection is

$$Y_{bus} = \begin{bmatrix} -j125 & 0 & j100 & 0 \\ 0 & -j6.25 & 0 & j5 \\ j100 & 0 & -j89 & j9 \\ 0 & j5 & j9 & -j13 \end{bmatrix}$$

The script file **lfybus** can be used to compute the bus admittance matrix. We use the following statements.

```
clear
%                                              Line code
%          Bus bus  R      X    1/2 B  = 1 for lines
%          nl  nr  pu     pu     pu   >1 or <1 tr. tap at bus nl
linedata=[1   3   0.0  0.0125  0.0        0.8
          3   4   0.0  0.20    0.0        1
          3   4   0.0  0.25    0.0        1
          4   2   0.0  0.16    0.0        1.25];
lfybus                        % form the bus admittance matrix
Ybus                            % displays Ybus  on the screen
```

Run **ch6p9** to obtain the bus admittance matrix.

6.10. In the two-bus system shown in Figure 58, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ pu. A load of 150 MW and 50 Mvar is taken from bus 2. The line admittance is $y_{12} = 10\angle{-73.74°}$ pu on a base of 100 MVA. The expression for real and reactive power at bus 2 is given by

$$\begin{aligned} P_2 &= \ \ 10|V_2||V_1|\cos(106.26° - \delta_2 + \delta_1) + 10|V_2|^2\cos(-73.74°) \\ Q_2 &= -10|V_2||V_1|\sin(106.26° - \delta_2 + \delta_1) - 10|V_2|^2\sin(-73.74°) \end{aligned}$$

Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2{}^{(0)} = 0°$. Perform two iterations. Partial derivatives of P_2, and Q_2 with respect to $|V_2|$, and δ_2 are

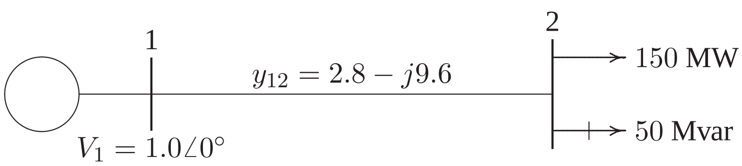


FIGURE 58
One-line diagram for Problem 6.10.

$$\begin{aligned} \frac{\partial P_2}{\partial \delta_2} &= 10|V_2||V_1|\sin(106.26° - \delta_2 + \delta_1) \\ \frac{\partial P_2}{\partial |V_2|} &= 10|V_1|\cos(106.26° - \delta_2 + \delta_1) + 20|V_2|\cos(-73.74°) \\ \frac{\partial Q_2}{\partial \delta_2} &= 10|V_2||V_1|\cos(106.26° - \delta_2 + \delta_1) \\ \frac{\partial Q_2}{\partial |V_2|} &= -10|V_1|\sin(106.26° - \delta_2 + \delta_1) - 20|V_2|\sin(-73.74°) \end{aligned}$$

The load expressed in per unit is

$$S_2^{sch} \;=\; -\frac{(150+j50)}{100} = -1.5 - j0.5 \;\; \text{pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\begin{aligned}
\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} &= -1.5 - [10\cos(106.26^\circ) + 10\cos(-73.74^\circ)] \\
&= -1.5 \;\; \text{pu} \\
\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} &= -0.5 - [-10\sin(106.26^\circ) - 10\sin(-73.74^\circ)] \\
&= -0.5 \;\; \text{pu}
\end{aligned}$$

The elements of the Jacobian matrix at the initial estimate are

$$\begin{aligned}
J_1^{(0)} &= 10(1)(1)\sin(106.26^\circ) = 9.6 \\
J_2^{(0)} &= 10(1)\cos(106.26^\circ) + 20(1)\cos(-73.74^\circ) = 2.8 \\
J_3^{(0)} &= 10(1)(1)\cos(106.26^\circ) = -2.8 \\
J_4^{(0)} &= -10(1)\sin(106.26^\circ) - 20(1)\sin(-73.74^\circ) = 9.6
\end{aligned}$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 9.6 & 2.8 \\ -2.8 & 9.6 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the first iteration is

$$\begin{aligned}
\Delta\delta_2^{(0)} &= -0.13 & \delta_2^{(1)} &= 0 + (-0.13) = -0.13 \;\; \text{radian} \\
\Delta|V_2^{(0)}| &= -0.09 & |V_2^{(1)}| &= 1 + (-0.09) = 0.91 \;\; \text{pu}
\end{aligned}$$

For the second iteration, we have

$$\begin{aligned}
\Delta P_2^{(1)} &= P_2^{sch} - P_2^{(1)} = -1.5 - (-1.3403) = -0.1597 \;\; \text{pu} \\
\Delta Q_2^{(1)} &= Q_2^{sch} - Q_2^{(1)} = -0.5 - (-0.3822) = -0.1178 \;\; \text{pu}
\end{aligned}$$

Also, computing the elements of the Jacobian matrix, the set of linear equations in the second iteration becomes

$$\begin{bmatrix} -0.1597 \\ -0.1178 \end{bmatrix} = \begin{bmatrix} 8.332 & 1.0751 \\ -3.659 & 8.3160 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the second iteration is

$$\begin{array}{ll} \Delta\delta_2^{(1)} = -0.0164 & \delta_2^{(2)} = -0.13 + (-0.0164) = -0.1464 \text{ radian} \\ \Delta|V_2^{(1)}| = -0.0214 & |V_2^{(2)}| = 0.91 + (-0.0214) = 0.8886 \text{ pu} \end{array}$$

6.11. In the two-bus system shown in Figure 59, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ pu. A load of 100 MW and 50 Mvar is taken from bus 2. The line impedance is $z_{12} = 0.12 + j0.16$ pu on a base of 100 MVA. Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2{}^{(0)} = 0°$. Perform two iterations.

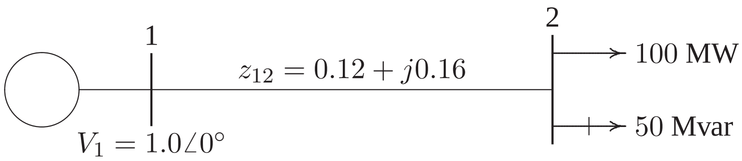


FIGURE 59
One-line diagram for Problem 6.11.

The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$
$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

The bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13° & 5\angle 126.87° \\ 5\angle 126.87° & 5\angle -53.13° \end{bmatrix}$$

Substituting for admittances, the expression for real and reactive power at bus 2 becomes

$$P_2 = \ 5|V_2||V_1| \cos(126.87° - \delta_2 + \delta_1) + 5|V_2|^2 \cos(-53.13°)$$
$$Q_2 = -5|V_2||V_1| \sin(126.87° - \delta_2 + \delta_1) - 5|V_2|^2 \sin(-53.13°)$$

Partial derivatives of P_2, and Q_2 with respect to $|V_2|$, and δ_2 are

$$\frac{\partial P_2}{\partial \delta_2} = 5|V_2||V_1| \sin(126.87° - \delta_2 + \delta_1)$$

$$\frac{\partial P_2}{\partial |V_2|} = 5|V_1|\cos(126.87^\circ - \delta_2 + \delta_1) + 10|V_2|\cos(-53.13^\circ)$$
$$\frac{\partial Q_2}{\partial \delta_2} = 5|V_2||V_1|\cos(126.87^\circ - \delta_2 + \delta_1)$$
$$\frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87^\circ - \delta_2 + \delta_1) - 10|V_2|\sin(-73.74^\circ)$$

The load expressed in per units is

$$S_2^{sch} = -\frac{(100 + j50)}{100} = -1.0 - j0.5 \text{ pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -1.0 - [5\cos(126.87^\circ) + 5\cos(-53.13^\circ)]$$
$$= -1.0 \text{ pu}$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -0.5 - [-5\sin(126.87^\circ) - 5\sin(-53.13^\circ)]$$
$$= -0.5 \text{ pu}$$

The elements of the Jacobian matrix at the initial estimate are

$$J_1^{(0)} = 5(1)(1)\sin(126.87^\circ) = 4$$
$$J_2^{(0)} = 5(1)\cos(126.87^\circ) + 10(1)\cos(-53.13^\circ) = 3$$
$$J_3^{(0)} = 5(1)(1)\cos(126.87^\circ) = -3$$
$$J_4^{(0)} = -5(1)\sin(126.87^\circ) - 10(1)\sin(-53.13^\circ) = 4$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the first iteration is

$$\Delta\delta_2^{(0)} = -0.10 \qquad \delta_2^{(1)} = 0 + (-0.10) = -0.10 \text{ radian}$$
$$\Delta|V_2^{(0)}| = -0.2 \qquad |V_2^{(1)}| = 1 + (-0.2) = 0.8 \text{ pu}$$

For the second iteration, we have

$$\Delta P_2^{(1)} = P_2^{sch} - P_2^{(1)} = -1.0 - (-0.7875) = -0.2125 \text{ pu}$$
$$\Delta Q_2^{(1)} = Q_2^{sch} - Q_2^{(1)} = -0.5 - (-0.3844) = -0.1156 \text{ pu}$$

Also, computing the elements of the Jacobian matrix, the set of linear equations in the second iteration becomes

$$\begin{bmatrix} -0.2125 \\ -0.1156 \end{bmatrix} = \begin{bmatrix} 2.9444 & 1.4157 \\ -2.7075 & 2.7195 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the second iteration is

$$\begin{aligned} \Delta\delta_2^{(1)} &= -0.0350 & \delta_2^{(2)} &= -0.1 + (-0.0350) = -0.135 \;\text{ radian} \\ \Delta|V_2^{(1)}| &= -0.0773 & |V_2^{(2)}| &= 0.8 + (-0.0773) = 0.7227 \;\text{ pu} \end{aligned}$$

6.12. Figure 60 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0°$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

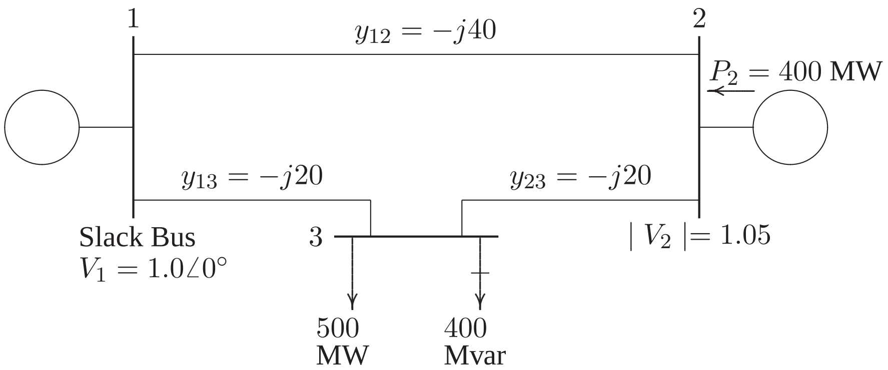


FIGURE 60
One-line diagram for problem 6.12.

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$\begin{aligned} P_2 &= 40|V_2||V_1|\cos(90° - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90° - \delta_2 + \delta_3) \\ P_3 &= 20|V_3||V_1|\cos(90° - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90° - \delta_3 + \delta_2) \\ Q_3 &= -20|V_3||V_1|\sin(90° - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90° - \delta_3 + \delta_2) + 40|V_3|^2 \end{aligned}$$

(b) Using Newton-Raphson method, start with the initial estimates of ${V_2}^{(0)} = 1.05 + j0$ and ${V_3}^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the

phasor values of V_2 and V_3. Perform two iterations.

(c) Check the power flow solution for Problem 6.12 using the **lfnewton** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

By inspection, the bus admittance matrix in polar form is

$$Y_{bus} = \begin{bmatrix} 60\angle -\frac{\pi}{2} & 40\angle\frac{\pi}{2} & 20\angle\frac{\pi}{2} \\ 40\angle\frac{\pi}{2} & 60\angle -\frac{\pi}{2} & 20\angle\frac{\pi}{2} \\ 20\angle\frac{\pi}{2} & 20\angle\frac{\pi}{2} & 40\angle -\frac{\pi}{2} \end{bmatrix}$$

(a) The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$
$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Substituting the elements of the bus admittance matrix in the above equations for P_2, P_3, and Q_3 will result in the given equations.

(b) Elements of the Jacobian matrix are obtained by taking partial derivatives of the given equations with respect to δ_2, δ_3 and $|V_3|$.

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_2||V_1|\sin(\frac{\pi}{2} - \delta_2 + \delta_1) + 20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$
$$\frac{\partial P_2}{\partial \delta_3} = -20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$
$$\frac{\partial P_2}{\partial |V_3|} = 20|V_2|\cos(\frac{\pi}{2} - \delta_2 + \delta_3)$$
$$\frac{\partial P_3}{\partial \delta_2} = -20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial P_3}{\partial \delta_3} = 20|V_3||V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial P_3}{\partial |V_3|} = 20|V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial Q_3}{\partial \delta_2} = -20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial Q_3}{\partial \delta_3} = 20|V_3||V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -20|V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) - 20|V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2) + 80|V_3|$$

The load and generation expressed in per units are

$$P_2^{sch} = \frac{400}{100} = 4.0 \text{ pu}$$
$$S_3^{sch} = -\frac{(500 + j400)}{100} = -5.0 - j4.0 \text{ pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_2| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 4.0 - (0) = 4.0$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = -5.0 - (0) = -5.0$$
$$\Delta Q_3^{(0)} = Q_3^{sch} - Q_3^{(0)} = -4.0 - (-1.0) = -3.0$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\begin{aligned} \Delta\delta_2^{(0)} &= 0.0275 & \delta_2^{(1)} &= 0 + 0.0275 = 0.0275 \text{ radian} = 1.5782^\circ \\ \Delta\delta_3^{(0)} &= -0.1078 & \delta_3^{(1)} &= 0 + (-0.1078) = -0.1078 \text{ radian} = -6.1790^\circ \\ \Delta|V_3^{(0)}| &= -0.0769 & |V_3^{(1)}| &= 1 + (-0.0769) = 0.9231 \text{ pu} \end{aligned}$$

For the second iteration, we have

$$\begin{bmatrix} 0.2269 \\ -0.3965 \\ -0.5213 \end{bmatrix} = \begin{bmatrix} 61.1913 & -19.2072 & 2.8345 \\ -19.2072 & 37.5615 & -4.9871 \\ 2.6164 & -4.6035 & 33.1545 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_3^{(1)}| \end{bmatrix}$$

and

$$\begin{aligned} \Delta\delta_2^{(1)} &= 0.0006 & \delta_2^{(2)} &= 0.0275 + 0.0006 = 0.0281 \text{ radian} = 1.61^\circ \\ \Delta\delta_3^{(1)} &= -0.0126 & \delta_3^{(2)} &= -0.1078 + (-0.0126) = -0.1204 \text{ radian} = -6.898^\circ \\ \Delta|V_3^{(1)}| &= -0.0175 & |V_3^{(2)}| &= 0.9231 + (-0.0175) = 0.9056 \text{ pu} \end{aligned}$$

(c) The power flow program **lfnewton** is used to obtain the solution, with the following statements:

```
clear
basemva = 100;  accuracy = 0.000001;  maxiter = 10;

%      Problem   6.12(c)
%      Bus  Bus Voltage Angle -Load---   -Generator-- Injected
%       No code Mag. Degree MW  MVAR  MW  MVAR Qmin Qmax  Mvar
busdata=[1   1  1.0   0.0  0.0  0.0   0.0  0.0   0   0    0
         2   2  1.05  0.0    0    0   400  0.0  600  0    0
         3   0  1.0   0.0  500  400   0.0  0.0   0   0    0];

%                                    Line code
%        Bus bus  R    X    1/2 B    = 1 for lines
%         nl  nr pu   pu      pu     >1 or <1 tr. tap at bus nl
linedata=[1   2  0.0  0.025   0.0        1
          1   3  0.0  0.05    0.0        1
          2   3  0.0  0.05    0.0        1];
disp('Problem 6.12(c)')
lfybus                        % form the bus admittance matrix
lfnewton           % Power flow solution by Gauss-Seidel method
busout           % Prints the power flow solution on the screen
lineflow       % Computes and displays the line flow and losses
```

The above statements are saved in the file **ch6p12c.m**. Run the program to obtain the solution.

6.13. For Problem 6.12:

(a) Obtain the power flow solution using the fast decoupled algorithm. Perform two iterations.
(b)Check the power flow solution for Problem 6.12 using the **decouple** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

(a) In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ form the bus admittance matrix in Problem 6.12 is

$$B' = \begin{bmatrix} -60 & 20 \\ 20 & -40 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.02 & -0.01 \\ -0.01 & -0.03 \end{bmatrix}$$

The expressions for real power at bus 2 and 3 and the reactive power at bus 3 are given in Problem 6.12. The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_3| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\begin{aligned}
\Delta P_2^{(0)} &= P_2^{sch} - P_2^{(0)} = 4 - (0) = 4 \\
\Delta P_3^{(0)} &= P_3^{sch} - P_3^{(0)} = -5 - (0) = -5 \\
\Delta Q_3^{(0)} &= Q_3^{sch} - Q_3^{(0)} = -4 - (-1) = -3
\end{aligned}$$

The fast decoupled power flow algorithm given by (6.77) becomes

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = - \begin{bmatrix} -0.02 & -0.0 \\ -0.01 & -0.03 \end{bmatrix} \begin{bmatrix} \frac{4}{1.05} \\ \frac{-5}{1.0} \end{bmatrix} = \begin{bmatrix} .0262 \\ -0.1119 \end{bmatrix}$$

Since bus 2 is a regulated bus, the corresponding row and column of $\boldsymbol{B'}$ are eliminated and we get

$$B'' = [-40]$$

From (6.78), we have

$$\Delta|V_3| = -\left[\frac{-1}{40}\right]\left[\frac{-3}{1.0}\right] = -0.075$$

The new bus voltages in the first iteration are

$$\begin{aligned}
\Delta\delta_2^{(0)} &= 0.0262 & \delta_2^{(1)} &= 0 + (0.0262) = 0.0262 \text{ radian} = 1.5006^\circ \\
\Delta\delta_3^{(0)} &= -0.1119 & \delta_3^{(1)} &= 0 + (-0.1119) = -0.1119 \text{ radian} = -6.4117^\circ \\
\Delta|V_3^{(0)}| &= -0.075 & |V_2^{(1)}| &= 1 + (-0.075) = 0.925 \text{ pu}
\end{aligned}$$

For the second iteration, the power residuals are

$$\begin{aligned}
\Delta P_2^{(1)} &= P_2^{sch} - P_2^{(1)} = 4 - (3.7739) = 0.2261 \\
\Delta P_3^{(1)} &= P_3^{sch} - P_3^{(1)} = -5 - (-4.7399) = -.2601 \\
\Delta Q_3^{(1)} &= Q_3^{sch} - Q_3^{(1)} = -4 - (-3.3994) = -0.6006
\end{aligned}$$

$$\begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.0 \\ -0.01 & -0.03 \end{bmatrix} \begin{bmatrix} \frac{0.2261}{1.05} \\ \frac{-0.2601}{0.925} \end{bmatrix} = \begin{bmatrix} 0.0015 \\ -0.0063 \end{bmatrix}$$

From (6.78), we have

$$\Delta|V_3| = -\left[\frac{-1}{40}\right]\left[\frac{-0.6006}{0.925}\right] = -0.0162$$

The new bus voltages in the second iteration are

$$\begin{aligned} \Delta\delta_2^{(1)} &= 0.0015 \qquad \delta_2^{(2)} = 0.0262 + (0.0015) = 0.0277 \text{ radian} = 1.5863^\circ \\ \Delta\delta_3^{(1)} &= -0.0063 \quad \delta_3^{(2)} = -0.1119 + (-0.0063) = -0.1182 \text{ radian} = -6.7716^\circ \\ \Delta|V_3^{(1)}| &= -0.0162 \quad |V_3^{(2)}| = 0.925 + (-0.0162) = 0.9088 \text{ pu} \end{aligned}$$

(c) The power flow program **decouple** is used to obtain the solution. The statements are the same as in Problem 6.12, except the **lfnewton** is replaced by **decouple**. The statements are saved in the file **ch6p13b**. Run this program to obtain the power flow solution.

6.14. The 26-bus power system network of an electric utility company is shown in Figure 61. Obtain the power flow solution by the following methods:
(a) Gauss-Seidel power flow (see Example 6.9).
(b) Newton-Raphson power flow (see Example 6.11).
(c) Fast decoupled power flow (see Example 6.13).

The load data is as follows.

LOAD DATA					
Bus	Load		Bus	Load	
No.	MW	Mvar	No.	MW	Mvar
1	51.0	41.0	14	24.0	12.0
2	22.0	15.0	15	70.0	31.0
3	64.0	50.0	16	55.0	27.0
4	25.0	10.0	17	78.0	38.0
5	50.0	30.0	18	153.0	67.0
6	76.0	29.0	19	75.0	15.0
7	0.0	0.0	20	48.0	27.0
8	0.0	0.0	21	46.0	23.0
9	89.0	50.0	22	45.0	22.0
10	0.0	0.0	23	25.0	12.0
11	25.0	15.0	24	54.0	27.0
12	89.0	48.0	25	28.0	13.0
13	31.0	15.0	26	40.0	20.0

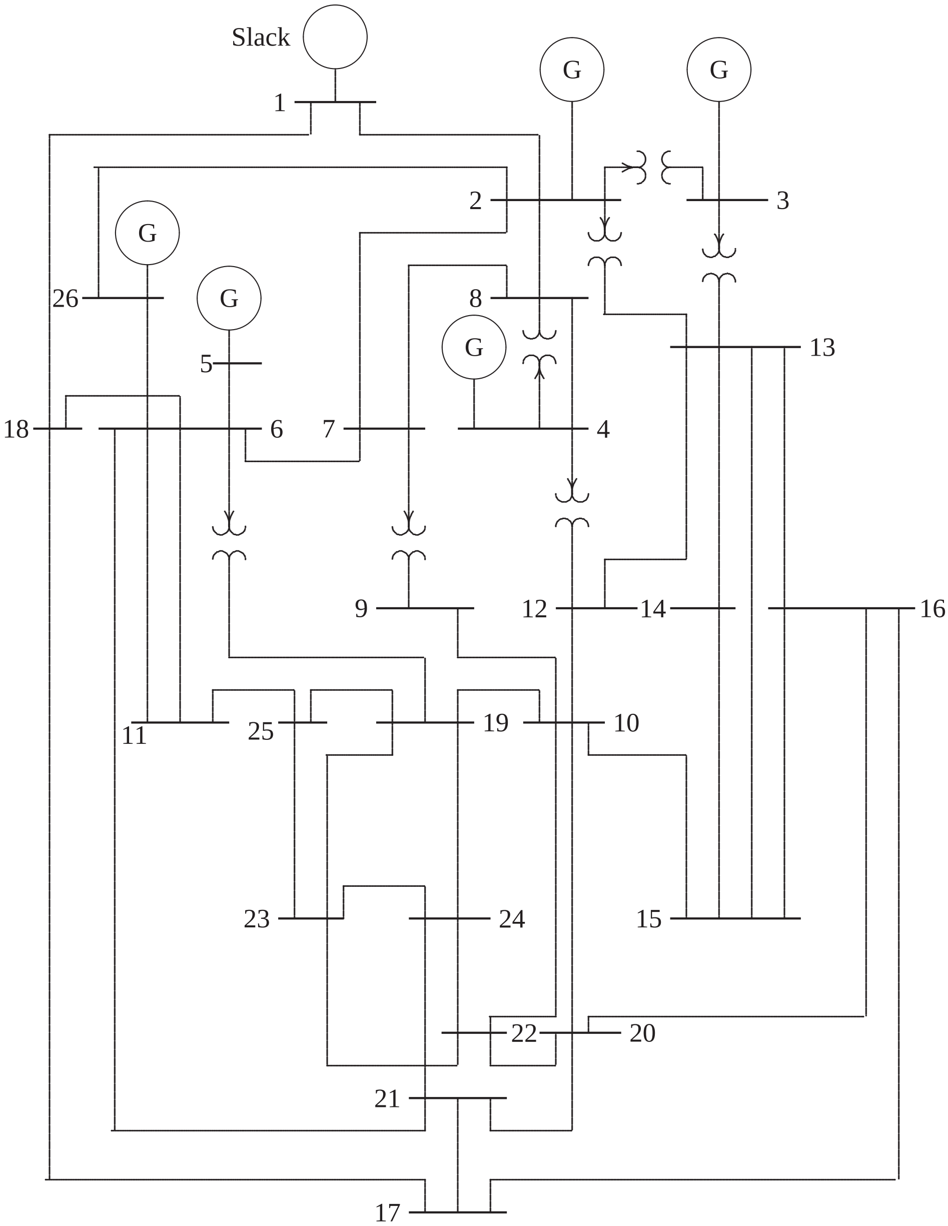


FIGURE 61
One-line diagram for Problem 6.14.

Voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.025\angle 0°$, is taken as the slack bus.

GENERATION DATA				
Bus No.	Voltage Mag.	Generation MW	Mvar Limits Min.	Mvar Limits Max.
1	1.025			
2	1.020	79.0	40.0	250.0
3	1.025	20.0	40.0	150.0
4	1.050	100.0	40.0	80.0
5	1.045	300.0	40.0	160.0
26	1.015	60.0	15.0	50.0

The Mvar of the shunt capacitors installed at substations and the transformer tap settings are given below.

SHUNT CAPACITORS	
Bus No.	Mvar
1	4.0
4	2.0
5	5.0
6	2.0
11	1.5
12	2.0
15	0.5
19	5.0

TRANSFORMER TAP	
Designation	Tap Setting
2 – 3	0.960
2 – 13	0.960
3 – 13	1.017
4 – 8	1.050
4 – 12	1.050
6 – 19	0.950
7 – 9	0.950

The line and transformer data containing the series resistance and reactance in per unit and one-half the total capacitance in per unit susceptance on a 100-MVA base are tabulated below.

LINE AND TRANSFORMER DATA									
Bus No.	Bus No.	R, pu	X, pu	$\frac{1}{2}B$, pu	Bus No.	Bus No.	R, pu	X, pu	$\frac{1}{2}B$, pu
1	2	0.0005	0.0048	0.0300	10	22	0.0069	0.0298	0.005
1	18	0.0013	0.0110	0.0600	11	25	0.0960	0.2700	0.010
2	3	0.0014	0.0513	0.0500	11	26	0.0165	0.0970	0.004
2	7	0.0103	0.0586	0.0180	12	14	0.0327	0.0802	0.000
2	8	0.0074	0.0321	0.0390	12	15	0.0180	0.0598	0.000
2	13	0.0035	0.0967	0.0250	13	14	0.0046	0.0271	0.001
2	26	0.0323	0.1967	0.0000	13	15	0.0116	0.0610	0.000
3	13	0.0007	0.0054	0.0005	13	16	0.0179	0.0888	0.001
4	8	0.0008	0.0240	0.0001	14	15	0.0069	0.0382	0.000
4	12	0.0016	0.0207	0.0150	15	16	0.0209	0.0512	0.000
5	6	0.0069	0.0300	0.0990	16	17	0.0990	0.0600	0.000
6	7	0.0053	0.0306	0.0010	16	20	0.0239	0.0585	0.000
6	11	0.0097	0.0570	0.0001	17	18	0.0032	0.0600	0.038
6	18	0.0037	0.0222	0.0012	17	21	0.2290	0.4450	0.000
6	19	0.0035	0.0660	0.0450	19	23	0.0300	0.1310	0.000
6	21	0.0050	0.0900	0.0226	19	24	0.0300	0.1250	0.002
7	8	0.0012	0.0069	0.0001	19	25	0.1190	0.2249	0.004
7	9	0.0009	0.0429	0.0250	20	21	0.0657	0.1570	0.000
8	12	0.0020	0.0180	0.0200	20	22	0.0150	0.0366	0.000
9	10	0.0010	0.0493	0.0010	21	24	0.0476	0.1510	0.000
10	12	0.0024	0.0132	0.0100	22	23	0.0290	0.0990	0.000
10	19	0.0547	0.2360	0.0000	22	24	0.0310	0.0880	0.000
10	20	0.0066	0.0160	0.0010	23	25	0.0987	0.1168	0.000

The power flow data and the command required are as follows.

```
clear
basemva = 100;  accuracy = 0.0001; accel = 1.6;  maxiter = 60;

%        Bus Bus Voltage Angle --Load-- --Generator----Injected
%        No  code Mag. Degree  MW   Mvar MW   Mvar Qmin Qmax Mvar
busdata=[1  1    1.025  0     51   41    0   0     0    0    4
         2  2    1.020  0     22   15   79   0    40  250    0
         3  2    1.025  0     64   50   20   0    40  150    0
         4  2    1.050  0     25   10  100   0    40   80    2
         5  2    1.045  0     50   30  300   0    40  160    5
         6  0    1.00   0     76   29    0   0     0    0    2
         7  0    1.00   0      0    0    0   0     0    0    0
```

```
         8  0    1.00   0     0    0    0   0     0    0     0
         9  0    1.00   0    89   50    0   0     0    0     3
        10  0    1.00   0     0    0    0   0     0    0     0
        11  0    1.00   0    25   15    0   0     0    0   1.5
        12  0    1.00   0    89   48    0   0     0    0     2
        13  0    1.00   0    31   15    0   0     0    0     0
        14  0    1.00   0    24   12    0   0     0    0     0
        15  0    1.00   0    70   31    0   0     0    0   0.5
        16  0    1.00   0    55   27    0   0     0    0     0
        17  0    1.00   0    78   38    0   0     0    0     0
        18  0    1.00   0   153   67    0   0     0    0     0
        19  0    1.00   0    75   15    0   0     0    0     5
        20  0    1.00   0    48   27    0   0     0    0     0
        21  0    1.00   0    46   23    0   0     0    0     0
        22  0    1.00   0    45   22    0   0     0    0     0
        23  0    1.00   0    25   12    0   0     0    0     0
        24  0    1.00   0    54   27    0   0     0    0     0
        25  0    1.00   0    28   13    0   0     0    0     0
        26  2    1.015  0    40   20   60   0    15   50     0];
%                                                 Line code
%         Bus bus   R        X       1/2 B  1 for line codes or
%         nl  nr    pu       pu       pu      tap setting value
linedata=[1   2   0.00055  0.00480  0.03000    1
          1  18   0.00130  0.01150  0.06000    1
          2   3   0.00146  0.05130  0.05000    0.96
          2   7   0.01030  0.05860  0.01800    1
          2   8   0.00740  0.03210  0.03900    1
          2  13   0.00357  0.09670  0.02500    0.96
          2  26   0.03230  0.19670  0.00000    1
          3  13   0.00070  0.00548  0.00050    1.017
          4   8   0.00080  0.02400  0.00010    1.050
          4  12   0.00160  0.02070  0.01500    1.050
          5   6   0.00690  0.03000  0.09900    1
          6   7   0.00535  0.03060  0.00105    1
          6  11   0.00970  0.05700  0.00010    1
          6  18   0.00374  0.02220  0.00120    1
          6  19   0.00350  0.06600  0.04500    0.95
          6  21   0.00500  0.09000  0.02260    1
          7   8   0.00120  0.00693  0.00010    1
          7   9   0.00095  0.04290  0.02500    0.95
          8  12   0.00200  0.01800  0.02000    1
          9  10   0.00104  0.04930  0.00100    1
         10  12   0.00247  0.01320  0.01000    1
         10  19   0.05470  0.23600  0.00000    1
         10  20   0.00660  0.01600  0.00100    1
```

```
         10  22   0.00690  0.02980  0.00500    1
         11  25   0.09600  0.27000  0.01000    1
         11  26   0.01650  0.09700  0.00400    1
         12  14   0.03270  0.08020  0.00000    1
         12  15   0.01800  0.05980  0.00000    1
         13  14   0.00460  0.02710  0.00100    1
         13  15   0.01160  0.06100  0.00000    1
         13  16   0.01793  0.08880  0.00100    1
         14  15   0.00690  0.03820  0.00000    1
         15  16   0.02090  0.05120  0.00000    1
         16  17   0.09900  0.06000  0.00000    1
         16  20   0.02390  0.05850  0.00000    1
         17  18   0.00320  0.06000  0.03800    1
         17  21   0.22900  0.44500  0.00000    1
         19  23   0.03000  0.13100  0.00000    1
         19  24   0.03000  0.12500  0.00200    1
         19  25   0.11900  0.22490  0.00400    1
         20  21   0.06570  0.15700  0.00000    1
         20  22   0.01500  0.03660  0.00000    1
         21  24   0.04760  0.15100  0.00000    1
         22  23   0.02900  0.09900  0.00000    1
         22  24   0.03100  0.08800  0.00000    1
         23  25   0.09870  0.11680  0.00000   1];
lfybus                      % Forms the bus admittance matrix
lfgauss        % Power flow solution by Newton-Raphson method
%lfnewton      % Power flow solution by Newton-Raphson method
%decouple      % Power flow solution by Newton-Raphson method
busout         % Prints the power flow solution on the screen
lfybus                       % form the bus admittance matrix
lineflow     % Computes and displays the line flow and losses
% (b)                   Comment lfgauss and uncomment lfnewton
% (c)     Comment lfgauss and lfnewton and uncomment decouple
```

Uncomment **lfgauss**, **lfnewton**, or **decouple** in turn, and run to obtain the power flow solution by these methods.

CHAPTER 7 PROBLEMS

7.1. Find a rectangle of maximum perimeter that can be inscribed in a circle of unit radius given by

$$g(x, y) = x^2 + y^2 - 1 = 0$$

Check the eigenvalues for sufficient conditions.

Consider point c with coordinates (x, y) as shown in Figure 62.

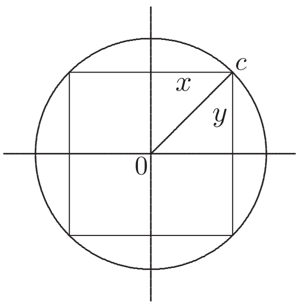


FIGURE 62
Constraint function of Problem 7.1.

The problem is to find the minimum value of the function

$$f(x, y) = 4(x + y)$$

subject to the equality constraint

$$g(x, y) = x^2 + y^2 - 1 = 0$$

Forming the Lagrangian function, we obtain

$$\mathcal{L} = 4(x+y) + \lambda(x^2 + y^2 - 1)$$

The resulting necessary conditions for constrained local maxima of $\mathcal{L}$ are

$$\frac{\partial \mathcal{L}}{\partial x} = 4 + 2\lambda x = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = 4 + 2\lambda y = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x^2 + y^2 - 1 = 0$$

From the first two conditions, we obtain $x = y$. Substituting for y in the third condition results in

$$2x^2 = 1$$

or

$$x = y = 0.707 \qquad \text{and} \quad \lambda = -\frac{4}{2(0.707)} = -2.828$$

and the perimeter is

$$p = 4(x+y) = 4(0.707 + 0.707) = 5.656$$

To see if the perimeter is a maximum, we evaluate the second derivatives and form the Hessian matrix

$$H = \begin{bmatrix} 2\lambda & 0 \\ 0 & 2\lambda \end{bmatrix} = \begin{bmatrix} -5.656 & 0 \\ 0 & -5.656 \end{bmatrix}$$

The second partial derivatives are negative, thus p is a maximum.

7.2. Find the minimum of the function

$$f(x,y) = x^2 + 2y^2$$

subject to the equality constraint

$$g(x,y) = x + 2y + 4 = 0$$

Check for the sufficient conditions.

Forming the Lagrangian function, we obtain

$$\mathcal{L} = x^2 + 2y^2 + \lambda(x + 2y + 4)$$

The resulting necessary conditions for constrained local minima of $\mathcal{L}$ are

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = 4y + 2\lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + 2y + 4 = 0$$

From the first two conditions, we obtain $x = y$. Substituting for y in the third condition results in

$$3x = -4$$

or

$$x = y = -\frac{4}{3} \qquad \text{and} \quad \lambda = -2x = \frac{8}{3}$$

and the minimum distance is

$$f(x, y) = x^2 + 2y^2 = \left(\frac{-4}{3}\right)^2 + 2\left(\frac{-4}{3}\right)^2 = 5.333$$

To see if this distance is a minimum, we evaluate the second derivatives and form the Hessian matrix

$$H = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

The second partial derivatives are positive, thus the minimum distance to origin is at point $(\hat{x}, \hat{y}) = (-\frac{4}{3}, -\frac{4}{3})$.

7.3. Use the Lagrangian multiplier method for solving constrained parameter optimization problems to determine an isosceles triangle of maximum area that may be inscribed in a circle of radius 1.

Consider point c with coordinates $(x, 1 + y)$ as shown in Figure 63. The problem is to maximize the area of the triangle given by

$$f(x, y) = x(1 + y)$$

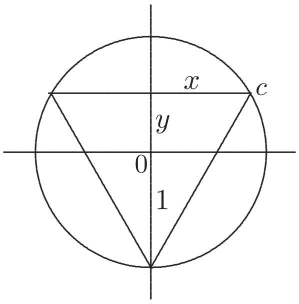


FIGURE 63
Constraint function of Problem 7.3.

subject to the equality constraint

$$g(x, y) = x^2 + y^2 - 1 = 0$$

Forming the Lagrangian function, we obtain

$$\mathcal{L} = x(1 + y) + \lambda(x^2 + y^2 - 1)$$

The resulting necessary conditions for constrained local maxima of $\mathcal{L}$ are

$$\frac{\partial \mathcal{L}}{\partial x} = 1 + y + 2\lambda x = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = x + 2\lambda y = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x^2 + y^2 - 1 = 0$$

From the first two conditions, we obtain

$$x^2 = y(1 + y)$$

Substituting for x^2 in the third condition results in

$$2y^2 + y - 1 = 0$$

which results in $y = 0.5$, and $y = -1$. For $y = -1$, $x = 0$, and area is zero. However, for $y = 0.5$, we obtain $x = 0.866$, and $\lambda = -0.866$, and the area is

$$f(\hat{x}, \hat{y}) = x(1 + y) = (0.866)(1.5) = 1.299$$

7.4. For a second-order bandpass filter with transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

determine the values of the damping ratio and natural frequency, ζ and ω_n, corresponding to a Bode plot whose peak occurs at 7071.07 radians/sec and whose half-power bandwidth is 12,720.2 radians/sec.

The frequency response transfer function is

$$H(j\omega) = \frac{\omega_n^2}{-\omega_n^2 + j2\zeta\omega_n\omega + \omega_n^2}$$

or

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\left(\frac{\omega}{\omega_n}\right)}$$

The magnitude of the frequency response transfer function is

$$|H(j\omega)| = \frac{1}{\left[\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}$$

The amplitude is maximum when the denominator is minimum. Thus the problem is to minimize the function

$$f(\omega) = \left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2$$

$|H(j\omega)|$ is maximum when $\frac{df(\omega)}{d\omega} = 0$. This condition results in the peak resonant frequency given by

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2} \tag{P7.1}$$

At the one-half power frequency ω_B, we have

$$\frac{1}{\left(1 - \left(\frac{\omega_B}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega_B}{\omega_n}\right)^2} = \frac{1}{2}$$

Solving for $\left(\frac{\omega_B}{\omega_n}\right)^2$ results in

$$\left(\frac{\omega_B}{\omega_n}\right)^2 = \left(1 - 2\zeta^2\right) \pm \sqrt{(1 - 2\zeta^2)^2 + 1} \tag{P7.2}$$

From (P7.1), we have

$$\left(1-2\zeta^2\right)=\left(\frac{\omega_r}{\omega_n}\right)^2 \tag{P7.3}$$

Substituting from (P7.3) into (P7.2), and considering the positive term, results in

$$\left(\frac{\omega_B}{\omega_n}\right)^2=\left(\frac{\omega_r}{\omega_n}\right)^2+\sqrt{\left(\frac{\omega_r}{\omega_n}\right)^4+1}$$

From the above expression, we obtain

$$\omega_n^4=\omega_B^2\left(\omega_B^2-2\omega_r^2\right) \tag{P7.4}$$

Substituting for $\omega_r = 7,071.07$ rad/s, and $\omega_B = 12,720.2$ rad/s, results in $\omega_n = 10,000$ rad/s, and from (P7.3), we have

$$\left(1-2\zeta^2\right)=\left(\frac{7,071.07}{10,000}\right)^2$$

or

$$\zeta = 0.5$$

7.5. Find the minimum value of the function

$$f(x,y)=x^2+y^2$$

subject to the equality constraint

$$g(x,y)=x^2-6x-y^2+17=0$$

Forming the Lagrangian function, we obtain

$$\mathcal{L}=x^2+y^2+\lambda(x^2-6x-y^2+17)$$

The resulting necessary conditions for constrained local maxima of $\mathcal{L}$ are

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial x}&=2x+\lambda(2x-6)=0\\ \frac{\partial\mathcal{L}}{\partial y}&=2y-2\lambda y=0\\ \frac{\partial\mathcal{L}}{\partial\lambda}&=x^2-6x-y^2+17=0\end{aligned}$$

From the second condition $\lambda = 1$. Substituting in the first condition

$$2x + (1)(2x - 6) = 0 \quad \text{or} \quad x = 1.5$$

Substituting for x in the third condition results in $y = 3.20156$. Thus the minimum value of the function is

$$f(\hat{x}, \hat{y}) = (1.5)^2 + (3.20156)^2 = 12.5$$

7.6. Find the minimum value of the function

$$f(x, y) = x^2 + y^2$$

subject to one equality constraint

$$g(x, y) = x^2 - 5x - y^2 + 20 = 0$$

and one inequality constraint

$$u(x, y) = 2x + y \geq 6$$

The unconstrained cost function from (7.16) is

$$\mathcal{L} = x^2 + y^2 + \lambda(x^2 - 5x - y^2 + 20) + \mu(2x + y - 6)$$

The resulting necessary conditions for constrained local minima of $\mathcal{L}$ are

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda(2x - 5) + 2\mu = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = 2y - 2\lambda y + \mu = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x^2 - 5x - y^2 + 20 = 0$$
$$\frac{\partial \mathcal{L}}{\partial \mu} = 2x + y - 6 = 0$$

Eliminating μ from the first two equations result in

$$2x(\lambda + 1) + 4y(\lambda - 1) = 5\lambda$$

From the fourth condition, we have

$$y = 6 - 2x$$

Substituting for y in the above equation, yields

$$x = \frac{24 - 19\lambda}{10 - 6\lambda}$$

Now substituting for x in the previous equation, we get

$$y = \frac{12 + 2\lambda}{10 - 6\lambda}$$

Substituting for x and y in the third condition (equality constraint) results in an equation in terms of λ

$$\left(\frac{24 - 19\lambda}{10 - 6\lambda}\right)^2 - 5\left(\frac{24 - 19\lambda}{10 - 6\lambda}\right) - \left(\frac{12 + 2\lambda}{10 - 6\lambda}\right)^2 + 20 = 0$$

from which we have the following equation

$$507\lambda^2 - 1690\lambda + 1232 = 0$$

Roots of the above equation are $\lambda = \frac{88}{39}$ and $\lambda = \frac{14}{13}$. Substituting for these values of λ in the expression for x and y, the corresponding extrema are

$$\begin{array}{lll} (x, y) = (\frac{16}{3}, -\frac{14}{3}) & \text{for} \quad \lambda = \frac{88}{39}, & \mu = -11.73 \\ (x, y) = (1, 4) & \text{for} \quad \lambda = \frac{14}{13}, & \mu = \frac{8}{13} \end{array}$$

The minimum distance from the cost function is 17, located at point $(1, 4)$, and the maximum distance is 50.22 located at point $(5.333, -4.666)$.

7.7. The fuel-cost functions in \$/h for two 800 MW thermal plants are given by

$$\begin{aligned} C_1 &= 400 + 6.0P_1 + 0.004P_1^2 \\ C_2 &= 500 + \quad \beta P_2 + \qquad \gamma P_2^2 \end{aligned}$$

where P_1 and P_2 are in MW.

(a) The incremental cost of power λ is \$8/MWh when the total power demand is 550 MW. Neglecting losses, determine the optimal generation of each plant.

(b) The incremental cost of power λ is \$10/MWh when the total power demand is 1300 MW. Neglecting losses, determine the optimal generation of each plant.
(c) From the results of (a) and (b) find the fuel-cost coefficients β and γ of the second plant.

$$\frac{dC_1}{dP_1} = 6 + 0.008P_1 = \lambda$$
$$\frac{dC_2}{dP_2} = \beta + 2\gamma P_2 = \lambda$$

(a) For $\lambda = 8$, and $P_D = 550$ MW, we have

$$P_1 = \frac{8-6}{0.008} = 250 \text{ MW}$$
$$P_2 = P_D - P_1 = 500 - 250 = 300 \text{ MW}$$

(b) For $\lambda = 10$, and $P_D = 1300$ MW, we have

$$P_1 = \frac{10-6}{0.008} = 500 \text{ MW}$$
$$P_2 = P_D - P_1 = 1300 - 500 = 800 \text{ MW}$$

(c) The incremental cost of power for plant 2 are given by

$$\beta + 2\gamma(300) = 8$$
$$\beta + 2\gamma(800) = 10$$

Solving the above equations, we find $\beta = 6.8$, and $\gamma = 0.002$

7.8. The fuel-cost functions in \$/h for three thermal plants are given by

$$C_1 = 350 + 7.20P_1 + 0.0040P_1^2$$
$$C_2 = 500 + 7.30P_2 + 0.0025P_2^2$$
$$C_3 = 600 + 6.74P_3 + 0.0030P_3^2$$

where P_1, P_2, and P_3 are in MW. The governors are set such that generators share the load equally. Neglecting line losses and generator limits, find the total cost in \$/h when the total load is

(i) $P_D = 450$ MW
(ii) $P_D = 745$ MW
(iii) $P_D = 1335$ MW

(i) For $P_D = 450$ MW, $P_1 = P_2 = P_3 = \frac{450}{3} = 150$ MW. The total fuel cost is

$$C_t = 350 + 7.20(150) + 0.004(150)^2 + 500 + 7.3(150) + 0.0025(150)^2 + 600 + 6.74(150) + 0.003(150)^2 = 4,849.75 \ \ \$/\text{h}$$

(ii) For $P_D = 745$ MW, $P_1 = P_2 = P_3 = \frac{745}{3}$ MW. The total fuel cost is

$$C_t = 350 + 7.20\left(\frac{745}{3}\right) + 0.004\left(\frac{745}{3}\right)^2 + 500 + 7.3\left(\frac{745}{3}\right) + 0.0025\left(\frac{745}{3}\right)^2 + 600 + 6.74\left(\frac{745}{3}\right) + 0.003\left(\frac{745}{3}\right)^2 = 7,310.46 \ \ \$/\text{h}$$

(iii) For $P_D = 1335$ MW, $P_1 = P_2 = P_3 = 445$ MW. The total fuel cost is

$$C_t = 350 + 7.20(445) + 0.004(445)^2 + 500 + 7.3(445) + 0.0025(445)^2 + 600 + 6.74(445) + 0.003(445)^2 = 12,783.04 \ \ \$/\text{h}$$

7.9 Neglecting line losses and generator limits, determine the optimal scheduling of generation for each loading condition in Problem 7.8
(a) by analytical technique, using (7.33) and (7.31).
(b) using Iterative method. Start with an initial estimate of $\lambda = 7.5$ \$/MWh.
(c) find the savings in \$/h for each case compared to the costs in Problem 7.8 when the generators shared load equally.
Use the **dispatch** program to check your results.

(a) (i) For $P_D = 450$ MW, from (7.33), λ is found to be

$$\lambda = \frac{450 + \frac{7.2}{0.008} + \frac{7.3}{0.005} + \frac{6.74}{0.006}}{\frac{1}{0.008} + \frac{1}{0.005} + \frac{1}{0.006}} = \frac{450 + 3483.333}{491.666} = 8.0 \ \ \$/\text{MWh}$$

Substituting for λ in the coordination equation, given by (7.31), the optimal dispatch is

$$P_1 = \frac{8.0 - 7.2}{2(0.004)} = 100$$
$$P_2 = \frac{8.0 - 7.3}{2(0.0025)} = 140$$
$$P_3 = \frac{8.0 - 6.74}{2(0.003)} = 210$$

(a) (ii) For $P_D = 745$ MW, from (7.33), λ is found to be

$$\lambda = \frac{745 + 3483.333}{491.666} = 8.6 \ \ \$/\text{MWh}$$

Substituting for λ in the coordination equation, given by (7.31), the optimal dispatch is

$$P_1 = \frac{8.6 - 7.2}{2(0.004)} = 175$$
$$P_2 = \frac{8.6 - 7.3}{2(0.0025)} = 260$$
$$P_3 = \frac{8.6 - 6.74}{2(0.003)} = 310$$

(a) (iii) For $P_D = 1335$ MW, from (7.33), λ is found to be

$$\lambda = \frac{1335 + 3483.333}{491.666} = 9.8 \ \ \$/\text{MWh}$$

Substituting for λ in the coordination equation, given by (7.31), the optimal dispatch is

$$P_1 = \frac{9.8 - 7.2}{2(0.004)} = 325$$
$$P_2 = \frac{9.8 - 7.3}{2(0.0025)} = 500$$
$$P_3 = \frac{9.8 - 6.74}{2(0.003)} = 510$$

(b) For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 7.5$. From coordination equations, given by (7.31), P_1, P_2, and P_3 are

$$P_1^{(1)} = \frac{7.5 - 7.2}{2(0.004)} = 37.5000$$
$$P_2^{(1)} = \frac{7.5 - 7.3}{2(0.0025)} = 40.0000$$
$$P_3^{(1)} = \frac{7.5 - 6.74}{2(0.003)} = 126.6666$$

(i) $P_D = 450$ MW, the error ΔP from (7.39) is

$$\Delta P^{(1)} = 450 - (37.5 + 40 + 126.6666) = 245.8333$$

From (7.37)

$$\Delta\lambda^{(1)} = \frac{245.8333}{491.6666} = 0.5$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 7.5 + 0.5 = 8.0$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{8.0 - 7.2}{2(0.004)} = 100$$
$$P_2^{(2)} = \frac{8.0 - 7.3}{2(0.0025)} = 140$$
$$P_3^{(2)} = \frac{8.0 - 6.74}{2(0.003)} = 210$$

and

$$\Delta P^{(2)} = 450 - (100 + 140 + 210) = 0.0$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iterations.

(ii) $P_D = 745$ MW, the error ΔP from (7.39) is

$$\Delta P^{(1)} = 745 - (37.5 + 40 + 126.6666) = 540.8333$$

From (7.37)

$$\Delta\lambda^{(1)} = \frac{540.8333}{491.6666} = 1.1$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 7.5 + 1.1 = 8.6$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{8.6 - 7.2}{2(0.004)} = 175$$
$$P_2^{(2)} = \frac{8.6 - 7.3}{2(0.0025)} = 260$$
$$P_3^{(2)} = \frac{8.6 - 6.74}{2(0.003)} = 310$$

and

$$\Delta P^{(2)} = 745 - (175 + 260 + 130) = 0.0$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iterations.

(iii) $P_D = 1335$ MW, the error ΔP from (7.39) is

$$\Delta P^{(1)} = 1335 - (37.5 + 40 + 126.6666) = 1130.8333$$

From (7.37)

$$\Delta\lambda^{(1)} = \frac{1130.8333}{491.6666} = 2.3$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 7.5 + 2.3 = 9.8$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{9.8 - 7.2}{2(0.004)} = 325$$
$$P_2^{(2)} = \frac{9.8 - 7.3}{2(0.0025)} = 500$$
$$P_3^{(2)} = \frac{9.8 - 6.74}{2(0.003)} = 510$$

and

$$\Delta P^{(2)} = 1335 - (325 + 500 + 510) = 0.0$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iterations.

(c)(i) For $P_1 = 100$ MW, $P_2 = 140$ MW, and $P_3 = 210$ MW, the total fuel cost is

$$C_t = 350 + 7.20(100) + 0.004(100)^2 + 500 + 7.3(140) + 0.0025(140)^2 + 600 + 6.74(210) + 0.003(210)^2 = 4,828.70 \ \ \$/\text{h}$$

Compared to Problem 7.8 (i), when the generators shared load equally, the saving is 4,849.75 - 4,828,70 = 21.05 $/h.

(c)(ii) For $P_1 = 175$ MW, $P_2 = 260$ MW, and $P_3 = 310$ MW, the total fuel cost is

$$C_t = 350 + 7.20(175) + 0.004(175)^2 + 500 + 7.3(260) + 0.0025(260)^2 + \\ 600 + 6.74(310) + 0.003(310)^2 = 7,277.20 \ \ \$/\text{h}$$

Compared to Problem 7.8 (ii), when the generators shared load equally, the saving is 7,310.46 - 7,277.20 = 33.26 $/h.

(c)(iii) For $P_1 = 325$ MW, $P_2 = 500$ MW, and $P_3 = 510$ MW, the total fuel cost is

$$C_t = 350 + 7.20(325) + 0.004(325)^2 + 500 + 7.3(500) + 0.0025(500)^2 + \\ 600 + 6.74(510) + 0.003(510)^2 = 12,705.20 \ \ \$/\text{h}$$

Compared to Problem 7.8 (iii), when the generators shared load equally, the saving is 12,783.04 - 12,705.20 = 77.84 $/h.

To check the results we use the following commands

```
cost = [350  7.2   0.004
        500  7.3   0.0025
        600  6.74  0.003];
disp('(i) Pdt = 450 MW')
Pdt = 450;
dispatch
gencost
disp('(ii) Pdt = 745 MW')
Pdt = 745;
dispatch
gencost
disp('(iii) Pdt = 1335 MW')
Pdt = 1335;
dispatch
gencost
```

The result is

```
(i) Pdt = 450 MW
Incremental cost of delivered power (system lambda)=8.0 $/MWh
Optimal Dispatch of Generation:

  100.0000
  140.0000
```

```
  210.0000
Total generation cost =    4828.70 $/h

(ii) Pdt = 745 MW
Incremental cost of delivered power (system lambda)=8.6 $/MWh
Optimal Dispatch of Generation:

  175.0000
  260.0000
  310.0000
Total generation cost =    7277.20 $/h

(iii) Pdt = 1335 MW
Incremental cost of delivered power (system lambda)=9.80 $/MWh
Optimal Dispatch of Generation:

  325.0000
  500.0000
  510.0000
Total generation cost =   12705.20 $/h
```

7.10. Repeat Problem 7.9 (a) and (b), but this time consider the following generator limits (in MW)

$$122 \le P_1 \le 400$$
$$260 \le P_2 \le 600$$
$$50 \le P_3 \le 445$$

Use the **dispatch** program to check your results.

In Problem 7.9, in part (a) (i), the optimal dispatch are $P_1 = 100$ MW, $P_2 = 140$ MW, and $P_3 = 210$ MW. Since P_1 and P_2 are less that their lower limit, these plants are pegged at their lower limits. That is, $P_1 = 122$, and P_2 = 260 MW. Therefore, $P_3 = 450 - (122 + 260) = 68$ MW.

In Problem 7.9, in part (a) (ii), the optimal dispatch are $P_1 = 175$ MW, $P_2 = 260$ MW, and $P_3 = 310$ MW. which are within the plants generation limits.

In Problem 7.9, in part (a) (iii), the optimal dispatch are $P_1 = 325$ MW, $P_2 = 500$ MW, and $P_3 = 510$ MW. Since P_3 exceed its upper limit, this plant is pegged at $P_2 = 445$. Therefore, a load of $1335 - 445 = 890$ MW must be shared between plants 1 and 2, with equal incremental fuel cost give by

$$\lambda = \frac{890 + \frac{7.2}{0.008} + \frac{7.3}{0.005}}{+\frac{1}{0.008} + \frac{1}{0.005}}$$

$$= \frac{890 + 2360}{325} = 10 \ \ \$/\text{MWh}$$

Substituting for λ in the coordination equation, given by (7.31), the optimal dispatch is

$$P_1 = \frac{10 - 7.2}{2(0.004)} = 350$$
$$P_2 = \frac{10 - 7.3}{2(0.0025)} = 540$$

Since P_1 and P_2 are within their limits the above result is the optimal dispatch.

(b) For part (iii), The iterative method is as follows

In Problem 7.9 (b) part (iii) starting with an initial value of $\lambda^{(1)} = 7.5$, the optimal dispatch was obtained in two iterations as $P_1 = 325$ MW, $P_2 = 500$ MW, and $P_3 = 510$ MW, with $\lambda = 9.8$\$/MWh. Since P_3 exceed its upper limit, this plant is pegged at $P_3 = 445$ and is kept constant at this value. Thus, the new imbalance in power is

$$\Delta P^{(2)} = 1335 - (325 + 500 + 445) = 65$$

From (7.37)

$$\Delta\lambda^{(2)} = \frac{65}{\frac{1}{2(0.004)} + \frac{1}{2(0.0025)}} = \frac{65}{325} = 0.2$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 9.8 + 0.2 = 10$$

For the third iteration, we have

$$P_1^{(3)} = \frac{10 - 7.2}{2(0.004)} = 350$$
$$P_2^{(3)} = \frac{10 - 7.3}{2(0.0025)} = 540$$
$$P_3^{(3)} = 445$$

and

$$\Delta P^{(3)} = 1335 - (350 + 540 + 445) = 0.0$$

$\Delta P^{(3)} = 0$, and the equality constraint is met and P_1 and P_2 are within their limits.

The following commands can be used to obtain the optimal dispatch of generation including generator limits.

To check the results, we use the following commands

```
cost = [350  7.2   0.004
        500  7.3   0.0025
        600  6.74  0.003];

mwlimits =[ 122 400
            260 600
            50 445]
%Pdt = 450;
%Pdt = 745;
Pdt = 1335;
dispatch
gencost
```

The result is

```
Incremental cost of delivered power (system lambda) = 10.0 $/MWh
Optimal Dispatch of Generation:

   350
   540
   445

Total generation cost =   12724.38 $/h
```

7.11. The fuel-cost function in \$/h of two thermal plants are

$$C_1 = 320 + 6.2P_1 + 0.004P_1^2$$
$$C_2 = 200 + 6.0P_2 + 0.003P_2^2$$

where P_1 and P_2 are in MW. Plant outputs are subject to the following limits (in MW)

$$50 \le P_1 \le 250$$
$$50 \le P_2 \le 350$$

The per-unit system real power loss with generation expressed in per unit on a 100-MVA base is given by

$$P_{L(pu)} = 0.0125P_{1(pu)}^2 + 0.00625P_{2(pu)}^2$$

The total load is 412.35 MW. Determine the optimal dispatch of generation. Start with an initial estimate of $\lambda = 7$ \$/MWh. Use the **dispatch** program to check your results.

In the cost function P_i is expressed in MW. Therefore, the real power loss in terms of MW generation is

$$P_L = \left[0.0125\left(\frac{P_1}{100}\right)^2 + 0.00625\left(\frac{P_2}{100}\right)^2\right] \times 100 \ \text{ MW}$$
$$= 0.000125P_1^2 + 0.0000625P_2^2 \ \text{ MW}$$

For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 7.0$. From coordination equations, given by (7.70), $P_1^{(1)}$, and $P_2^{(1)}$ are

$$P_1^{(1)} = \frac{7.0 - 6.2}{2(0.004 + 7.0 \times 0.000125)} = 82.05128 \quad \text{MW}$$
$$P_2^{(1)} = \frac{7.0 - 6.0}{2(0.003 + 7.0 \times 0.0000625)} = 145.4545 \ \text{ MW}$$

The real power loss is

$$P_L^{(1)} = 0.000125(82.05228)^2 + 0.0000625(145.4545)^2 = 2.16386$$

Since $P_D = 412.35$ MW, the error $\Delta P^{(1)}$ from (7.68) is

$$\Delta P^{(1)} = 412.35 + 2.16386 - (82.05128 + 145.4545) = 187.008$$

From (7.71)

$$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.0 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2(0.003 + 7.0 \times 0.0000625)^2}$$
$$= 243.2701$$

From (7.65)

$$\Delta\lambda^{(1)} = \frac{187.008}{243.2701} = 0.7687$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 7.0 + 0.7687 = 7.7687$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{7.7687 - 6.2}{2(0.004 + 7.7687 \times 0.000125)} = 157.7848 \quad \text{MW}$$
$$P_2^{(2)} = \frac{7.7687 - 6.0}{2(0.003 + 7.7687 \times 0.0000625)} = 253.7229 \;\; \text{MW}$$

The real power loss is

$$P_L^{(2)} = 0.000125(157.7848)^2 + 0.0000625(253.7229)^2 = 7.1355$$

Since $P_D = 412.35$ MW, the error $\Delta P^{(2)}$ from (7.68) is

$$\Delta P^{(2)} = 412.35 + 7.1355 - (157.7848 + 253.7229) = 7.9778$$

From (7.71)

$$\sum_{i=1}^{2} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.7687 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2(0.003 + 7.7687 \times 0.0000625)^2}$$
$$= 235.514$$

From (7.65)

$$\Delta\lambda^{(2)} = \frac{7.9778}{235.514} = 0.0339$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 7.7687 + 0.0339 = 7.8026$$

For the third iteration, we have

$$P_1^{(3)} = \frac{7.8026 - 6.2}{2(0.004 + 7.8026 \times 0.000125)} = 161.0548 \quad \text{MW}$$
$$P_2^{(3)} = \frac{7.8026 - 6.0}{2(0.003 + 7.8026 \times 0.0000625)} = 258.4252 \;\; \text{MW}$$

The real power loss is

$$P_L^{(3)} = 0.000125(161.0548)^2 + 0.0000625(258.4252)^2 = 7.4163$$

Since $P_D = 412.35$ MW, the error $\Delta P^{(3)}$ from (7.68) is

$$\Delta P^{(3)} = 412.35 + 7.4163 - (161.0548 + 258.4252) = 0.2863$$

From (7.71)

$$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.8026 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2(0.003 + 7.8026 \times 0.0000625)^2}$$
$$= 235.18$$

From (7.65)

$$\Delta\lambda^{(3)} = \frac{0.2683}{235.18} = 0.0012$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 7.8026 + 0.0011 = 7.8038$$

Continuing the process, the optimal dispatch after six iteration converges to $P_1 = 161.1765$ MW, and $P_2 = 258.6003$ MW, with an incremental production cost of $\lambda = 7.8038$\$/MWh.

To check the results, we use the following commands

We use the following commands

```
cost = [320  6.2   0.004
        200  6.0   0.003];
mwlimits =[ 50  250
            50  350];
Pdt = 412.35;
B=[0.0125   0
   0     0.00625]
basemva=100;
lambda = 7.0;
dispatch
gencost
```

The result is

```
B =

    0.0125         0
         0    0.0063

Incremental cost of delivered power (system lambda)= 7.803862 $/MWh
Optimal Dispatch of Generation:

  161.1765
  258.6003
Total generation cost =    3375.43 $/h
```

7.12. The 9-bus power system network of an Electric Utility Company is shown in Figure 64. The load data is tabulated below. Voltage magnitude, generation schedule and the reactive power limits for the regulated buses are also tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.03\angle 0°$, is taken as the slack bus.

LOAD DATA		
Bus	Load	
No.	MW	Mvar
1	0	0
2	20	10
3	25	15
4	10	5
5	40	20
6	60	40
7	10	5
8	80	60
9	100	80

GENERATION DATA				
Bus	Voltage	Generation	Mvar Limits	
No.	Mag.	MW	Min.	Max.
1	1.03			
2	1.04	80	0	250
7	1.01	120	0	100

The Mvar of the shunt capacitors installed at substations are given below

SHUNT CAPACITORS	
Bus No.	Mvar
3	1.0
4	3.0

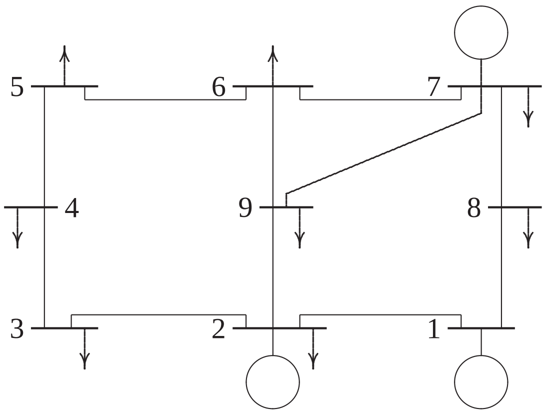


FIGURE 64
One-line diagram for Problem 7.12.

The line data containing the series resistance and reactance in per unit, and one-half of the total capacitance in per unit susceptance on a 100 MVA base is tabulated below.

LINE DATA				
Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}$ B, PU
1	2	0.018	0.054	0.0045
1	8	0.014	0.036	0.0030
2	9	0.006	0.030	0.0028
2	3	0.013	0.036	0.0030
3	4	0.010	0.050	0.0000
4	5	0.018	0.056	0.0000
5	6	0.020	0.060	0.0000
6	7	0.015	0.045	0.0038
6	9	0.002	0.066	0.0000
7	8	0.032	0.076	0.0000
7	9	0.022	0.065	0.0000

The generator's operating costs in \$/h are as follows:

$$C_1 = 240 + 6.7P_1 + 0.009P_1^2$$
$$C_2 = 220 + 6.1P_2 + 0.005P_2^2$$
$$C_7 = 240 + 6.5P_7 + 0.008P_7^2$$

The generator's real power limits are

GENERATOR REAL POWER LIMITS		
Gen.	Min. MW	Max. MW
1	50	200
2	50	200
7	50	100

Write the necessary commands to obtain the optimal dispatch of generation using **dispatch**. Continue the optimization process until the difference (absolute value) between the scheduled slack generation, determined from the coordination equation, and the slack generation, obtained from the power flow solution, is within 0.001 MW.

Place the following statements in an m-file, and run to obtain the optimal solution.

```
clear
basemva = 100;  accuracy = 0.00001;  maxiter = 15;

%       Bus Bus Voltage Angle -Load-  -----Generator--Injected
```

```
%       No code Mag.  Degree  MW  MVAR MW MVAR Qmin Qmax  Mvar
busdata=[1  1   1.03   0.0     0    0    0   0     0   50   0
         2  2   1.04   0.0    20   10   80   0     0  250   0
         3  0   1.00   0.0    25   15    0   0     0    0   1
         4  0   1.00   0.0    10    5    0   0     0    0   3
         5  0   1.00   0.0    40   20    0   0     0    0   0
         6  0   1.00   0.0    60   40    0   0     0    0   0
         7  2   1.01   0.0    10    5  120   0     0  100   0
         8  0   1.00   0.0    80   60    0   0     0    0   0
         9  0   1.00   0.0   100   80    0   0     0    0   0];
%                                        Line code
%         Bus bus   R     X      1/2 B   1 for line code or
%         nl  nr  p.u.   p.u.    p.u.    tap setting value
linedata=[1   2   0.018  0.054   0.0045     1
          1   8   0.014  0.036   0.0030     1
          2   9   0.006  0.030   0.0028     1
          2   3   0.013  0.036   0.0030     1
          3   4   0.010  0.050   0.0000     1
          4   5   0.018  0.056   0.0000     1
          5   6   0.020  0.060   0.0000     1
          6   7   0.015  0.045   0.0038     1
          6   9   0.002  0.066   0.0000     1
          7   8   0.032  0.076   0.0000     1
          7   9   0.022  0.065   0.0000     1];
cost=[240   6.7    0.009
      220   6.1    0.005
      240   6.5    0.008];
mwlimits=[50    200
          50    200
          50    100];
lfybus                     % form the bus admittance matrix
lfnewton     % Power flow solution by Newton-Raphson method
busout       % Prints the power flow solution on the screen
%lineflow
bloss               % Obtains the loss formula coefficients
gencost
dispatch
while dpslack > 0.001
lfnewton
bloss
dispatch
end
busout
gencost
lineflow
```

CHAPTER 8 PROBLEMS

8.1. A sinusoidal voltage given by $v(t) = 390\sin(315t + \alpha)$ is suddenly applied to a series RL circuit. $R = 32\ \Omega$ and $L = 0.4$ H.
(a) The switch is closed at such a time as to permit no transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.
(b) The switch is closed at such a time as to permit maximum transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.
(c) What is the maximum value of current in part (b) and at what time does this occur after the switch is closed?

$$v(t) = 390\sin(315t + \alpha)$$
$$i(t) = I_m \sin(315t + \alpha - \gamma) - I_m e^{-t/\tau}\sin(\alpha - \gamma)$$

(a) For no transient $\alpha = \gamma$

$$\gamma = \tan^{-1}\frac{(315)(0.4)}{32} = 75.75^\circ \quad \Rightarrow \quad \alpha = 75.75^\circ$$
$$Z = 32 + j(315)(0.4) = 32 + j126 = 130\angle 75.75^\circ\ \ \Omega$$
$$I = \frac{390}{130} = 3\ \text{ A} \quad \Rightarrow \quad i(t) = 3\sin 315t$$

(b) For maximum transient current $\alpha - \gamma = -90^\circ$. Therefore, $\alpha = 75.75 - 90 = -14.25^\circ$, and $\tau = \frac{L}{R} = 0.0125$ sec, and the current is

$$i(t) = 3\sin(315t - \frac{\pi}{2}) + 3e^{-80t}$$

(c)

$$\frac{di(t)}{dt} = (3)(315)\cos(315t - \frac{\pi}{2}) - 240e^{-80t} = 0$$

Use the command `[Imax, k] = max(i), tmax = t(k)` to find the maximum value of current, and the corresponding time.

$$i_{max} = 4.371 \text{ A}$$
$$t_{max} = 0.0096 \text{ sec}$$

Save the following statements in a file named **ch8p1.m**, and run to obtain the plots.

```
disp('Problem 8.1')
R = 32; L = 0.4; tau= L/R;w=315; X=w*L;
Z=R+j*X; Vm = 390;
gama=angle(Z);
gamad=gama*180/pi;
Im=Vm/130;
disp('(a)')
alpha1= gama; alpha = alpha1*180/pi
t=0:.0001:.08;
i1=Im*sin(315*t+alpha1 -gama);
subplot(211), plot(t, i1), grid
xlabel('t, sec'), ylabel('i(t)')
[I1max,k] = max(i1)
tmax= t(k)
disp('(b), (c)')
alpha2=gama - pi/2;  alpha = alpha2*180/pi
i2=Im*sin(315*t+alpha2 -gama)-Im*exp(-t/tau).*sin(alpha2-gama);
subplot(212), plot(t, i2), grid
xlabel('t, sec'), ylabel('i(t)')
[I2max,k] = max(i2)
tmax= t(k)
```

8.2. Consider the synchronous generator in Example 8.2. A three-phase short circuit is applied at the instant when the rotor direct axis position is at $\delta = 30°$. Use **ode45** to simulate (8.36), and obtain and plot the transient waveforms for the current in each phase and the field current.

In Example 8.2, set $\delta = 30°$, rename the file and run the program.

8.3. Consider the synchronous generator in Example 8.2. A line-to-line short circuit occurs between phases b and c at the instant when the rotor direct axis position

is at $\delta = 30°$. Use **ode45** to simulate (8.46), and obtain the transient waveforms for the current in phase b and the field current.

In Example 8.3, set $\delta = 30°$, rename the file and run the program.

8.4. Consider a line-to-ground short circuit between phase a and ground in a synchronous generator. Apply the short circuit conditions

$$\begin{aligned} v_a &= 0 \\ i_b &= i_c = 0 \end{aligned}$$

to the voltage equation of the synchronous machine given by (8.4). Substitute for all the flux linkages in terms of the inductances given by (8.9)–(8.13) and verify Equation (8.47).

Substituting the given conditions into (8.4), and (8.7) result in

$$\begin{bmatrix} v_a \\ -v_F \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r_F & 0 & 0 \\ 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \frac{d}{dt}\begin{bmatrix} \lambda_a \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix}$$

$$\begin{bmatrix} \lambda_a \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{aF} & L_{aD} & L_{aQ} \\ L_{Fa} & L_F & M_R & 0 \\ L_{Da} & M_R & L_D & 0 \\ L_{Qa} & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

Substituting the derivative of the second equation into the first one yields

$$\begin{bmatrix} v_a \\ -v_F \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r + \dot{L}_{aa} & \dot{L}_{aF} & \dot{L}_{aD} & \dot{L}_{aQ} \\ \dot{L}_{Fa} & r_F & 0 & 0 \\ \dot{L}_{Da} & 0 & r_D & 0 \\ \dot{L}_{Qa} & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \begin{bmatrix} L_{aa} & L_{aF} & L_{aD} & L_{aQ} \\ L_{Fa} & L_F & M_R & 0 \\ L_{Da} & M_R & L_D & 0 \\ L_{Qa} & 0 & 0 & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

The above inductances are defined in (8.9)–(8.13). Their derivatives are

$$\begin{aligned} \dot{L}_{aa} &= -2\omega L_m \sin 2\theta \\ \dot{L}_{aF} &= -\omega M_F \sin\theta \\ \dot{L}_{aD} &= -\omega M_D \sin\theta \\ \dot{L}_{aQ} &= -\omega M_Q \cos\theta \end{aligned}$$

Substituting for the inductances, we obtain

$$\begin{bmatrix} 0 \\ -v_F \\ 0 \\ 0 \end{bmatrix} =$$

$$-\begin{bmatrix} r - 2\omega L_m \sin 2\theta & -\omega M_F \sin\theta & -\omega M_D \sin\theta & \omega M_Q \cos\theta \\ -\omega M_F \sin\theta & r_F & 0 & 0 \\ -\omega M_D \sin\theta & 0 & r_D & 0 \\ \omega M_Q \cos\theta & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

$$-\begin{bmatrix} L_s + L_m \cos 2\theta & M_F \cos\theta & M_D \cos\theta & M_Q \sin\theta \\ M_F \cos\theta & L_F & M_R & 0 \\ M_D \cos\theta & M_R & L_D & 0 \\ M_Q \sin\theta & 0 & 0 & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

where

$$L_s = \frac{1}{3}(L_0 + L_d + L_q)$$
$$L_m = \frac{1}{3}(L_d - L_q)$$

8.5. Consider the synchronous generator in Example 8.2. A line-to-ground short circuit occurs between phase a and ground at the instant when the rotor direct axis position is at $\delta = 30°$. Use **ode45** to simulate (8.47), and obtain the transient waveforms for the current in phase a and the field current.

In Example 8.4, set $\delta = 30°$, rename the file and run the program.

8.6. A three-phase, 60-Hz synchronous machine is driven at constant synchronous speed by a prime mover. The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The machine has the following per unit reactances and time constants.

$$X''_d = 0.25 \text{ pu} \qquad \tau''_d = 0.04 \text{ sec}$$
$$X'_d = 0.45 \text{ pu} \qquad \tau'_d = 1.4 \text{ sec}$$
$$X_d = 1.50 \text{ pu}$$

(a) Determine the steady state, transient, and subtransient short circuit currents.
(b) Obtain and plot the fundamental-frequency waveform of the armature current for a three-phase short circuit at the terminals of the generator. Assume the short circuit is applied at the instant when the rotor direct axis is along the magnetic axis

of phase a, i.e., $\delta = 0$.

(a)

$$I_d = \frac{E_0}{X_d} = \frac{1.0}{1.5} = 0.6667 \text{ pu}$$
$$I'_d = \frac{E_0}{X'_d} = \frac{1.0}{0.45} = 2.2222 \text{ pu}$$
$$I''_d = \frac{E_0}{X''_d} = \frac{1.0}{0.25} = 4.0 \text{ pu}$$

(b) From (8.57), we have

$$i_{ac}(t) = \sqrt{2}(1)\left[\left(\frac{1}{0.25} - \frac{1}{0.45}\right)e^{-t/0.04} + \left(\frac{1}{0.45} - \frac{1}{1.5}\right)e^{-t/1.4} + \frac{1}{1.5}\right]\sin\omega t$$
$$i_{ac}(t) = \left[2.5142e^{-25t} + 2.2e^{-0.7143t} + 0.9428\right]\sin\omega t$$

Use the following statements to obtain the short-circuit plot.

```
disp('Problem 8.6')
w0 = 2*pi*60; E0 = 1.0; delta = 0;
Xd2dash = 0.25;   Xddash  = 0.45;  Xd = 1.5;
tau2dash = 0.04; taudash = 1.4;
t=0:1/(4*240):.34;
iac = sqrt(2)*E0*((1/Xd2dash-1/Xddash)*exp(-t/tau2dash) +...
      (1/Xddash-1/Xd)*exp(-t/taudash) + 1/Xd).*sin(w0*t + delta);
di1 = sqrt(2)*E0*((1/Xd2dash-1/Xddash)*exp(-t/tau2dash) +...
      (1/Xddash-1/Xd)*exp(-t/taudash) + 1/Xd);
di2 = sqrt(2)*E0*((1/Xddash-1/Xd)*exp(-t/taudash)+1/Xd);
di3=sqrt(2)*E0/Xd*ones(1,length(t));
plot(t, iac, t, di1, t, di2, t, di3), grid
xlabel('t, sec'), ylabel('iac, A')
[t' iac']
```

8.7. For the machine in Problem 8.6, assume that a three-phase short circuit is applied at the instant when the rotor quadrature axis is along the magnetic axis of phase a, i.e., $\delta = \pi/2$ radians. Obtain and plot the asymmetrical waveform of the armature current for phase a. The armature short circuit time constant is $\tau_a = 0.3$ sec.

From (8.59), we have

$$I_{dc} = \sqrt{2}\frac{E_0}{X''_d}\sin\delta\; e^{-t/\tau_a}$$
$$= \sqrt{2}\frac{1}{0.25}(1)e^{-3.3333t} = 5.6568e^{-3.3333t}$$

The asymmetrical short circuit current is

$$i_{asy}(t) = \left[2.5142e^{-25t} + 2.2e^{-0.7143t} + 0.9428\right]\sin(\omega t + \frac{\pi}{2}) + 5.6568e^{-3.3333t}$$

8.8. A three-phase, 60-Hz synchronous machine is driven at constant synchronous speed by a prime mover. The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The generator is suddenly subjected to a symmetrical three-phase short circuit at the instant when direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$. An oscillogram of the short-circuited current is obtained. The peak values at the first two cycles, at the 20th and 21st cycles, and the steady value after a long time were recorded as tabulated in the following table.

I_{max}, pu	5.4016	4.6037	$\cdots$	2.6930	2.6720	$\cdots$	0.9445
Time, sec	0.0042	0.0208	$\cdots$	0.3208	0.3375	$\cdots$	10.004

Determine the transient and the subtransient reactances and time constants.

The following statements are written with reference to the procedure in Section (8.9).

```
disp('Problem 8.8')
E0 = 1.0;
Im=[5.4016   4.6037  2.6930  2.6720   0.9445];
 t=[0.0042   0.0208  0.3208  0.3375   10.004];
I = Im/sqrt(2);           % The rms value of the above envelop
id=I(5);               % rms value of the steady short-circuit
Dt2 =[t(3) t(4)];                             % First two cycles
Di2 = I(3)-id I(4)-id];%Diff. between transient envelop and id
LDi2= log(Di2);           %Natural log of the above two points
c2=polyfit(Dt2, LDi2, 1);%Finds coefficients of the polynomial
             % i.e. the slope and intercept of a straight line
iddash=(exp(c2(2))+id)    % rms value of the transient current
Xddash=E0/iddash             % Direct-axis transient reactance
taudash=abs(1/c2(1))   % Short-circuit transient time constant
Di=(iddash-id)*[exp(-t(1)/taudash)  exp(-t(2)/taudash)];
Di1=[I(1)-Di(1)-id   I(2)-Di(2)-id];    % Subtransient envelop
LDi1=log(Di1);
Dt1 =[t(1)  t(2)];        % Natural log of the first two points
c1=polyfit(Dt1, LDi1, 1);%Finds coefficients of the polynomial
             % i.e. the slope and intercept of a straight line
id2dash=exp(c1(2))+iddash   %rms value of subtransient current
Xd2dash= E0/id2dash       % Direct-axis subtransient reactance
tau2dash=abs(1/c1(1))%Short-circuit subtransient time constant
```

```
t=0:.005:.045;
fit2 = polyval(c2, t); % line C2 evaluated for all values of t
fit1 = polyval(c1, t);   line C1 evaluated for all values of t
plot(t, fit1, t, fit2),grid % Logarithmic plot of id'' and id'
ylabel('Ln(I), pu')  % intercepts are ln(Id'') and Ln(Id')
xlabel('t, sec')      % slopes are the reciprocal of the
                                          % time constants
```

The result is

$$\begin{array}{lll} I'_d = 2.2273 \text{ pu} & X'_d = 0.4490; \text{ pu} & \tau'_d = 1.3821 \text{ sec} \\ I''_d = 4.0024 \text{ pu} & X''_d = 0.2498; \text{ pu} & \tau''_d = 0.0397 \text{ sec} \end{array}$$

8.9. A 100-MVA, three-phase, 60-Hz generator driven at constant speed has the following per unit reactances and time constants

$$\begin{array}{ll} X''_d = 0.20 \text{ pu} & \tau''_d = 0.04 \text{ sec} \\ X'_d = 0.30 \text{ pu} & \tau'_d = 1.0 \text{ sec} \\ X_d = 1.20 \text{ pu} & \tau_a = 0.25 \text{ sec} \end{array}$$

The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The generator is suddenly subjected to a symmetrical three-phase short circuit at the instant when $\delta = \pi/2$. Obtain and plot the asymmetrical waveform of the armature current for phase a. Determine

(a) The rms value of the ac component in phase a at $t = 0$.
(b) The dc component of the current in phase a at $t = 0$.
(c) The rms value of the asymmetrical current in phase a at $t = 0$.

$$I_d = \frac{E_0}{X_d} = \frac{1.0}{1.2} = 0.8333 \text{ pu}$$
$$I'_d = \frac{E_0}{X'_d} = \frac{1.0}{0.3} = 3.3333 \text{ pu}$$
$$I''_d = \frac{E_0}{X''_d} = \frac{1.0}{0.2} = 5.0 \text{ pu}$$

From (8.57), we have

$$i_{ac}(t) = \sqrt{2}(1)\Big[(5 - 3.3333)e^{-t/0.04}\,(3.3333 - 0.8333)\,e^{-t} + 0.8333\Big]\sin(\omega t + \frac{\pi}{2})$$
$$i_{ac}(t) = \Big[2.3571e^{-25t} + 3.5355e^{-t} + 1.1785\Big]\sin(\omega t + \frac{\pi}{2})$$

From (8.59), we have

$$I_{dc} = \sqrt{2}(5)(1)e^{-4t} = 7.0711e^{-4t} \ \text{pu}$$

Use the following statements to obtain the short-circuit plot.

```
disp('Problem 8.9')
w0 = 2*pi*60; E0 = 1.0; delta = pi/2;
Xd2dash = 0.2;   Xddash  = 0.3;  Xd = 1.2;
tau2dash = 0.04; taudash = 1.0; taua=0.25;
t=0:1/(4*240):.34;
%t=1/240:1/60: 1;
iac = sqrt(2)*E0*((1/Xd2dash-1/Xddash)*exp(-t/tau2dash) +...
  (1/Xddash-1/Xd)*exp(-t/taudash) + 1/Xd).*sin(w0*t + delta);
iasy=iac+sqrt(2)*E0/Xd2dash*sin(delta)*exp(-t/taua);
plot(t, iasy), grid
xlabel('t, sec'), ylabel('iac, A')
[t' iasy']
```

(a) At $t = 0$, $\hat{i_{ac}} = 2.3571+3.5355+1.1785 = 7.0711$ pu. Therefore, $I_{ac\,rms} = 5$ pu.
(b) At $t = 0$, $I_{dc} = 7.011$
(c) At $t = 0$, the rms value of the asymmetrical current in phase a is

$$I_{asym} = \sqrt{5^2 + (7.0711)^2} = 8.66 \ \text{pu}$$

8.10. A 100-MVA, 20-kV, 60-Hz three-phase synchronous generator is connected to a 100-MVA, 20/400 kV three-phase transformer. The machine has the following per unit reactances and time constants.

$$X''_d = 0.15 \text{ pu} \qquad \tau''_d = 0.035 \text{ sec}$$
$$X'_d = 0.25 \text{ pu} \qquad \tau'_d = 0.50 \text{ sec}$$
$$X_d = 1.25 \text{ pu} \qquad \tau_a = 0.3 \text{ sec}$$

The transformer reactance is 0.25 per unit. The generator is operating at the rated voltage and no-load when a three-phase short circuit occurs at the secondary terminals of the transformer.
(a) Find the subtransient, transient, and the steady state short circuit currents in per unit and actual amperes on both sides of the transformer.
(b) What is the maximum asymmetrical rms current (ac plus dc) at the beginning of the short circuit?
(c) Obtain and plot the instantaneous expression for the short circuit current including the dc component. Assume $\delta = \pi/2$ radians.

(a) The base current on the generator side is

$$I_{B1} = \frac{S_B}{\sqrt{3}\,V_{B1}} = \frac{100 \times 10^3}{\sqrt{3}\,20} = 2886.75 \;\; \text{A}$$

The base current on the secondary side of the transformer is

$$I_{B2} = \frac{20}{400}(2886.75) = 144.3375 \;\; \text{A}$$

the subtransient, transient and the steady state short circuit currents are

$$\begin{aligned}
I_d'' &= \frac{1.0}{0.15+0.25} = 2.5 \text{ pu} = 7,216.88 \text{ A} \quad \text{on the generator side} \\
&\qquad\qquad\qquad\qquad = 360.84 \text{ A} \quad \text{on the 400-kV side} \\
I_d' &= \frac{1.0}{0.25+0.25} = 2.0 \;\; \text{pu} = 5.773.5 \text{ A} \quad \text{on the generator side} \\
&\qquad\qquad\qquad\qquad = 288.675 \text{ A} \quad \text{on the 400-kV side} \\
I_d &= \frac{1.0}{1.25+0.25} = 0.0.6667 \;\; \text{pu} = 1,924.5 \text{ A} \quad \text{on the generator side} \\
&\qquad\qquad\qquad\qquad = 96.225 \text{ A} \quad \text{on the 400-kV side}
\end{aligned}$$

(b) The maximum rms current (ac plus dc) at the beginning of the fault is

$$I_{asy} = \sqrt{3}\,I_d'' = \sqrt{3}\,(2.5) = 4.33 \text{ pu} = 12,500 \text{ A} \;\; \text{on the generator side}$$

(c) The instantaneous short circuit current including the dc component from (8.60), for $\delta = \pi/2$ is

$$i(t) = \sqrt{2}[(I_d'' - I_d')e^{-t/0.4} + (I_d' - I_d)e^{-t/1.1} + I_d]\sin(377t + \pi/2) + \sqrt{2}I_d''e^{-t/0.25}$$

or

$$i(t) = [0.7071e^{-28.57t} + 1.8856e^{-2t} + 0.9428]\sin(377t + \pi/2) + 3.5355e^{-3.333t} \;\; \text{pu}$$

Use the following commands to obtain the solution and a plot of $i(t)$.

```
disp('problem 8.10')
Xd = 1.25; Xd1d = 0.25; Xd2d = 0.15; Xt = 0.25;
SB = 100; VL1 = 20; VL2 = 400;
IB1 = SB*1000/(sqrt(3)*VL1);
IB2 = VL1/VL2*IB1;
Id2d = 1.0/(Xd2d+Xt);
disp(['Id2dash = ', num2str(Id2d) 'pu ',num2str(Id2d*IB1),...
      'A   ',num2str(Id2d*IB2), 'A'])
Id1d = 1.0/(Xd1d+Xt);
```

```
disp(['Id1dash = ', num2str(Id1d) 'pu ',num2str(Id1d*IB1),...
      'A   ',num2str(Id1d*IB2), 'A'])
Id = 1.0/(Xd+Xt);
disp(['Id =  ', num2str(Id) 'pu   ', num2str(Id*IB1),...
      'A   ',num2str(Id*IB2), 'A'])
Iasy = sqrt(3)*Id2d;
disp(['Iasy = ', num2str(Iasy) 'pu   ',num2str(Iasy*IB1),...
      'A       ',num2str(Iasy*IB2), 'A'])

t=0:.02:6;
it = sqrt(2)*( (Id2d-Id1d)*exp(-t/0.035) + (Id1d-Id)*exp...
   (-t/0.5)+Id ).*sin(377*t+pi/2)+sqrt(2)*Id2d*exp(-t/0.3);
plot(t, it), grid
xlabel('t, sec'), ylabel('i, A')
title('Short circuit current including dc component')
```

8.11. In Problem 8.10, a three-phase load of 80-MVA, 0.8 power factor lagging is connected to the transformer secondary side. The line-to-line voltage at the load terminals is 400 kV. A three-phase short circuit occurs at the load terminals. Find the generator transient current including the load current.

The load may be represented by a per unit impedance.

$$S_L = \frac{80\angle 36.87^\circ}{100} = 0.8\angle 36.87^\circ \;\; \text{pu}$$
$$V = \frac{400}{400} = 1\angle 0^\circ \;\; \text{pu}$$
$$Z_L = \frac{|V^2|}{S^*} = \frac{1}{0.8\angle -36.87^\circ} = 1.0 + j0.75 \;\; \text{pu}$$

Before fault, the load current is

$$I_L = \frac{V}{Z_L} = \frac{1.0\angle 0^\circ}{1.0 + j0.75} = 0.64 - j0.48 = 1\angle -36.87^\circ \;\; \text{pu}$$

The emf behind transient reactance is

$$E' = V + j(X'_d + X_t)I_L$$
$$= 1.0\angle 0^\circ + j(0.25 + 0.25)(0.64 - j0.48) = 1.24 + j0.32 = 1.2806\angle 14.47^\circ \text{ pu}$$

When the fault is applied by closing switch S, the generator short circuit transient current is

$$I'_g = \frac{E'}{j(X'_d + X_t)} = \frac{1.24 + j0.32}{j(0.25 + 0.25)} = 0.64 - j2.4 = 2.56\angle -75.53^\circ \;\; \text{pu}$$
$$= 7,393.69\angle -75.53^\circ \;\; \text{A}$$

8.12. A 100-MVA, 20-kV synchronous generator is connected through a transmission line to a 100-MVA, 20-kV synchronous motor. The per unit transient reactances of the generator and motor are 0.25 and 0.20, respectively. The line reactance on the base of 100 MVA is 0.1 per unit. The motor is taking 50 MW at 0.8 power factor leading at a terminal voltage of 20 kV. A three-phase short circuit occurs at the generator terminals. Determine the transient currents in each of the two machines and in the short circuit.

$$S_m = \frac{50}{0.8}\angle{-36.87^\circ} \text{ MVA} = 0.625\angle{-36.87^\circ} \text{ pu}$$
$$V_m = \frac{20\angle 0^\circ}{20} = 1.0\angle 0^\circ \text{ pu}$$
$$I = \frac{S_m^*}{V_m^*} = 0.625\angle 36.87^\circ \text{ pu}$$

The generator emf behind transient reactance is

$$E_g' = V_m + j(X_{dg}' + X_L)I = 1.0\angle 0^\circ + j(0.25 + 0.10)(0.5 + j0.375)$$
$$= 0.8688 + j0.1750 = 0.8862\angle 11.389^\circ \text{ pu}$$

The motor emf behind transient reactance is

$$E_m' = V_m - j(X_{dm}')I = 1.0\angle 0^\circ - j(0.20)(0.5 + j0.375)$$
$$= 1.075 - j0.10 = 1.0796\angle{-5.3145^\circ} \text{ pu}$$

$$I_g' = \frac{E_g'}{j(X_{dg}')} = \frac{0.8862\angle 11.389^\circ}{j0.25} = 0.7 - j3.475 = 3.545\angle{-78.61^\circ} \text{ pu}$$

$$I_m' = \frac{E_m'}{j(X_{dm}' + X_L)} = \frac{1.0796\angle{-5.3145^\circ}}{j0.30} = -0.3333 - j3.5833$$
$$= 3.5988\angle{-95.3145^\circ} \text{ pu}$$

The short-circuit current is

$$I_f = I_g' + I_m' = (0.70 - j3.475) + (-0.3333 - j3.58333)$$
$$= 7.0679\angle{-87.026^\circ} \text{ pu}$$

CHAPTER 9 PROBLEMS

9.1. The system shown in Figure 65 is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is $j160\ \Omega$. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the short-circuit current and the short-circuit MVA.

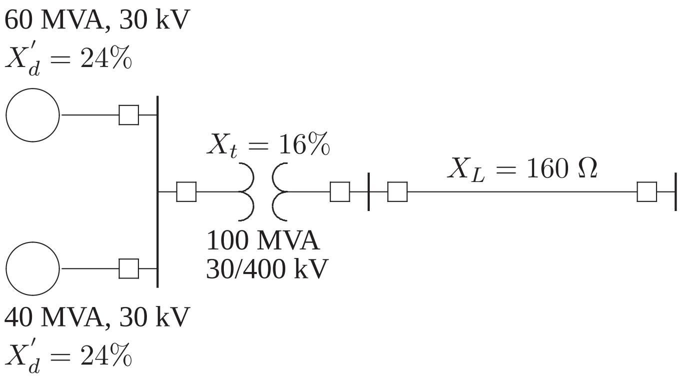


FIGURE 65
One-line diagram for Problem 9.1.

The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1,600\ \ \Omega$$

and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375 \text{ A}$$

The reactances on a common 100 MVA base are

$$X'_{dg1} = \frac{100}{60}(0.24) = 0.4 \text{ pu}$$
$$X'_{dg2} = \frac{100}{40}(0.24) = 0.6 \text{ pu}$$
$$X_t = \frac{100}{100}(0.16) = 0.16 \text{ pu}$$
$$X_{line} = \frac{160}{1600} = 0.1 \text{ pu}$$

The impedance diagram is as shown in Figure 66.

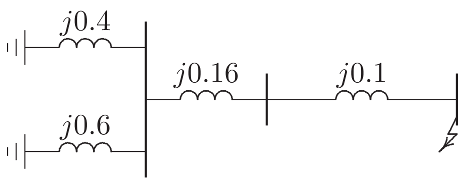


FIGURE 66
The impedance diagram for Problem 9.1.

Impedance to the point of fault is

$$X = j\frac{(0.4)(0.6)}{0.4+0.6} + j0.16 + j0.1 = j0.5 \text{ pu}$$

The fault current is

$$I_f = \frac{1}{j0.5} = 2\angle{-90^\circ} \text{ pu}$$
$$= (144.3375)(2\angle{-90^\circ}) = 288.675\angle{-90^\circ} \text{ A}$$

The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \text{ MVA}$$

9.2. The system shown in Figure 67 shows an existing plant consisting of a generator of 100 MVA, 30 kV, with 20 percent subtransient reactance and a generator of 50 MVA, 30 kV with 15 percent subtransient reactance, connected in parallel to a 30-kV bus bar. The 30-kV bus bar feeds a transmission line via the circuit breaker

C which is rated at 1250 MVA. A grid supply is connected to the station bus bar through a 500-MVA, 400/30-kV transformer with 20 percent reactance. Determine the reactance of a current limiting reactor in ohm to be connected between the grid system and the existing bus bar such that the short-circuit MVA of the breaker C does not exceed.

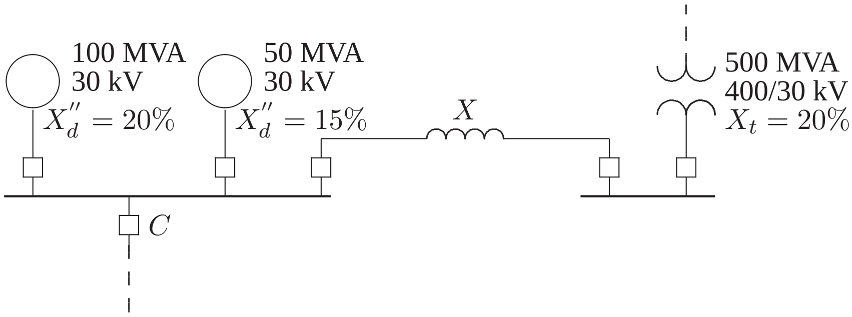


FIGURE 67
One-line diagram for Problem 9.2.

The base impedance for line is

$$Z_B = \frac{(30)^2}{100} = 9 \ \ \Omega$$

The reactances on a common 100 MVA base are

$$X''_{d\,g1} = \frac{100}{100}(0.2) = 0.2 \ \ \text{pu}$$
$$X''_{d\,g2} = \frac{100}{50}(0.15) = 0.3 \ \ \text{pu}$$
$$X_t = \frac{100}{500}(0.2) = 0.04 \ \ \text{pu}$$

The impedance diagram is as shown in Figure 68.
Reactance to the point of fault is

$$X_f = \frac{S_B}{\text{SCMVA}} = \frac{100}{1250} = 0.08 \ \ \text{pu}$$

Parallel reactance of the generators is

$$X_{||} = \frac{(0.2)(0.3)}{0.2 + 0.3} = 0.12 \ \ \text{pu}$$

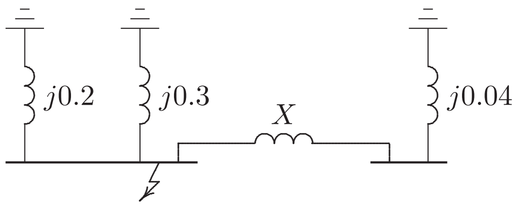


FIGURE 68
The impedance diagram for Problem 9.2.

From Figure 68, reactance to the point of fault is

$$\frac{(0.12)(X+0.04)}{0.12+(X+0.04)} = 0.08$$

Solving for X, we get $X = 0.2$ pu., or

$$X_{\Omega} = (X)(Z_B) = (0.2)(9) = 1.8 \;\; \Omega$$

9.3. The one-line diagram of a simple power system is shown in Figure 69. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

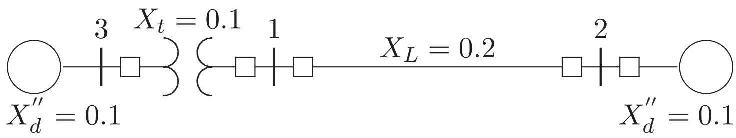


FIGURE 69
One-line diagram for Problem 9.3.

The impedance diagram is as shown in Figure 70.
(a) Impedance to the point of fault is

$$X = j\frac{(0.2)(0.3)}{0.2+0.3} = j0.12 \;\; \text{pu}$$

The fault current is

$$I_f = \frac{1}{j0.12 + j0.08} = 5\angle{-90^\circ} \;\; \text{pu}$$

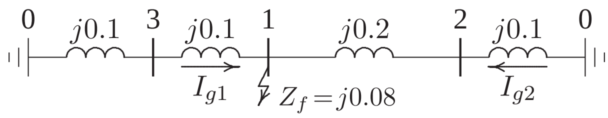


FIGURE 70
The impedance diagram for Problem 9.3.

(b)

$$V_1 = (j0.08)(-j5) = 0.4 \text{ pu}$$
$$I_{g1} = \frac{j0.3}{j0.5}(5)\angle{-90^\circ} = 3\angle{-90^\circ} \text{ pu}$$
$$I_{g2} = \frac{j0.2}{j0.5}(5)\angle{-90^\circ} = 2\angle{-90^\circ} \text{ pu}$$
$$V_2 = 0.4 + (j0.2)(-j2) = 0.8 \text{ pu}$$
$$V_3 = 0.4 + (j0.1)(-j3) = 0.7 \text{ pu}$$

9.4. The one-line diagram of a simple three-bus power system is shown in Figure 71 Each generator is represented by an emf behind the subtransient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

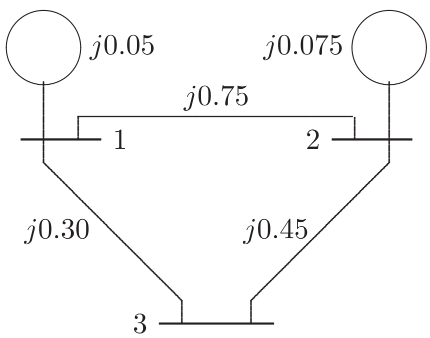


FIGURE 71
One-line diagram for Problem 9.4.

Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure 72(a), we have

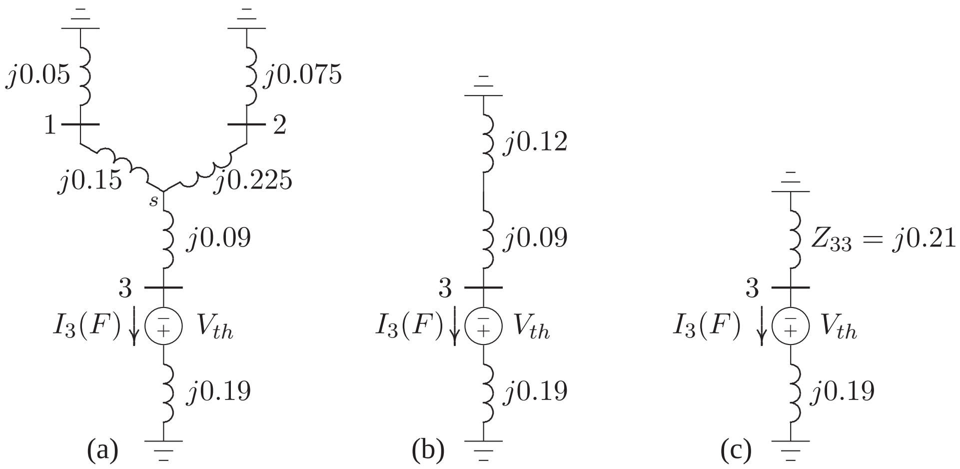


FIGURE 72
Reduction of Thévenin's equivalent network.

$$Z_{1s} = \frac{(j0.3)(j0.75)}{j1.5} = j0.15 \qquad Z_{2s} = \frac{(j0.75)(j0.45)}{j1.5} = j0.225$$

$$Z_{3s} = \frac{(j0.3)(j0.45)}{j1.5} = j0.09$$

Combining the parallel branches, Thévenin's impedance is

$$Z_{33} = \frac{(j0.2)(j0.3)}{j0.2 + j0.3} + j0.09$$
$$= j0.12 + j0.09 = j0.21$$

From Figure 72(c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

With reference to Figure 72(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.3}{j0.2 + j0.3} I_3(F) = -j1.5 \quad \text{pu}$$
$$I_{G2} = \frac{j0.2}{j0.2 + j0.3} I_3(F) = -j1.0 \quad \text{pu}$$

For the bus voltage changes from Figure 72(a), we get

$$\Delta V_1 = 0 - (j0.05)(-j1.5) \quad = -0.075 \;\; \text{pu}$$
$$\Delta V_2 = 0 - (j0.075)(-j1) \quad = -0.075 \;\; \text{pu}$$
$$\Delta V_3 = (j0.19)(-j2.5) - 1.0 = -0.525 \;\; \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.075 = 0.925 \text{ pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.075 = 0.925 \text{ pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.525 = 0.475 \text{ pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \quad \text{pu}$$

9.5. The one-line diagram of a simple four-bus power system is shown in Figure 73 Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A bolted three-phase fault occurs at bus 4.

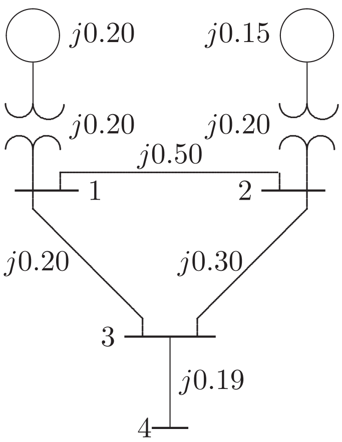


FIGURE 73
One-line diagram for Problem 9.5.

(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

(c) Repeat (a) and (b) for a fault at bus 2 with a fault impedance of $Z_f = j0.0225$.

(a) Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure 74(a), we have

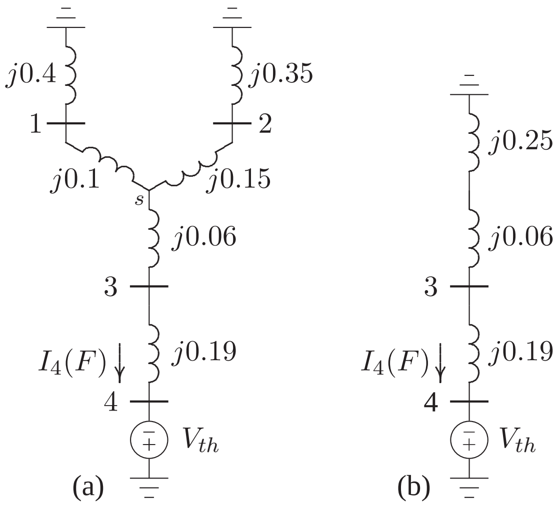


FIGURE 74
Reduction of Thévenin's equivalent network.

$$Z_{1s} = \frac{(j0.2)(j0.5)}{j1.0} = j0.10 \qquad Z_{2s} = \frac{(j0.5)(j0.3)}{j1.0} = j0.15$$
$$Z_{3s} = \frac{(j0.2)(j0.3)}{j1.0} = j0.06$$

Combining the parallel branches, Thévenin's impedance is

$$Z_{33} = \frac{(j0.5)(j0.5)}{j0.5 + j0.5} + j0.06 + j0.19 = j0.5$$

The fault current is

$$I_4(F) = \frac{V_4(F)}{Z_{44}} = \frac{1.0}{j0.5} = -j2.0 \quad \text{pu}$$

With reference to Figure 74(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \quad \text{pu}$$
$$I_{G2} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \quad \text{pu}$$

(b) For the bus voltage changes from Figure 74(a), we get

$$\begin{aligned}
\Delta V_1 &= 0 - (j0.4)(-j1.0) &&= -0.4 \ \text{ pu} \\
\Delta V_2 &= 0 - (j0.35)(-j1) &&= -0.35 \ \text{ pu} \\
\Delta V_3 &= 1 - (j0.19)(-j2) &&= -0.62 \ \text{ pu}
\end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned}
V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.4 = 0.60 \ \text{ pu} \\
V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.35 = 0.65 \ \text{ pu} \\
V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.62 = 0.38 \ \text{ pu} \\
V_4(F) &= 0 \ \text{ pu}
\end{aligned}$$

The short circuit-currents in the lines are

$$\begin{aligned}
I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.6}{j0.5} = 0.1\angle{-90^\circ} \ \text{ pu} \\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = 1.1\angle{-90^\circ} \ \text{ pu} \\
I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = 0.9\angle{-90^\circ} \ \text{ pu} \\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = 2.0\angle{-90^\circ} \ \text{ pu}
\end{aligned}$$

(c) (a) Combining parallel branches between buses 1 and 2 results in the circuit shown in Figure 75(a). Combining the parallel branches, Thévenin's impedance is

$$Z_{22} = \frac{(j0.65)(j0.35)}{j0.65 + j0.35} = j0.2275$$

The fault current is

$$I_2(F) = \frac{V_4(F)}{Z_{44} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4.0 \ \text{ pu}$$

With reference to Figure 75(a), the current divisions between the two generators are

$$\begin{aligned}
I_{G1} &= \frac{j0.35}{j0.65 + j0.35} I_2(F) = -j1.4 \ \text{ pu} \\
I_{G2} &= \frac{j0.65}{j0.65 + j0.35} I_2(F) = -j2.6 \ \text{ pu}
\end{aligned}$$

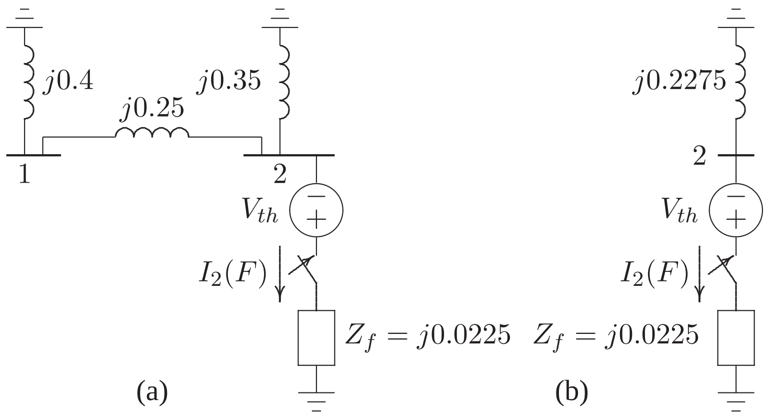


FIGURE 75
Reduction of Thévenin's equivalent network.

(c) (b) For the bus voltage changes from Figure 74(a), we get

$$\begin{aligned}
\Delta V_1 &= 0 - (j0.4)(-j1.4) \quad = -0.56 \text{ pu} \\
\Delta V_2 &= 0 - (j0.35)(-j2.6) \quad = -0.91 \text{ pu} \\
\Delta V_3 &= -j0.56 - (j0.2)(-\frac{j1.4}{2}) \quad = -0.70 \text{ pu} \\
\Delta V_4 &= \Delta V_3 = -0.70 \text{ pu}
\end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned}
V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.56 = 0.44 \text{ pu} \\
V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.91 = 0.09 \text{ pu} \\
V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.70 = 0.30 \text{ pu} \\
V_4(F) &= V_4(0) + \Delta V_4 = 1.0 - 0.70 = 0.30 \text{ pu}
\end{aligned}$$

The short circuit-currents in the lines are

$$\begin{aligned}
I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = 0.7\angle{-90^\circ} \text{ pu} \\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = 0.7\angle{-90^\circ} \text{ pu} \\
I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = 0.7\angle{-90^\circ} \text{ pu} \\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.3}{j0.19} = 0 \text{ pu}
\end{aligned}$$

9.6. Using the method of building algorithm find the bus impedance matrix for the network shown in Figure 76.

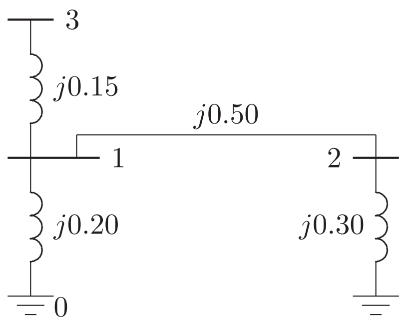


FIGURE 76
One-line diagram for Problem 9.6.

Add branch 1, $z_{10} = j0.2$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.20$$

Next, add branch 2, $z_{20} = j0.3$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.3 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.15$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.5$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.2 & 0 & j0.2 & -j0.2 \\ 0 & j0.3 & 0 & j0.3 \\ j0.2 & 0 & j0.35 & -j0.2 \\ -j0.2 & j0.3 & -j0.2 & Z_{44} \end{bmatrix} \end{aligned}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.5 + j0.2 + j0.3 - 2(j0) = j1.0$$

and

$$\begin{aligned}\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{j1.0}\begin{bmatrix} -j0.2 \\ j0.3 \\ -j0.2 \end{bmatrix}\begin{bmatrix} -j0.2 & j0.3 & -j0.2 \end{bmatrix} \\ &= \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix}\end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned}\mathbf{Z}_{bus} &= \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix} \\ &= \begin{bmatrix} j0.16 & j0.06 & j0.16 \\ j0.06 & j0.21 & j0.06 \\ j0.16 & j0.06 & j0.31 \end{bmatrix}\end{aligned}$$

9.7. Obtain the bus impedance matrix for the network of Problem 9.3.

Add branch 1, $z_{20} = j0.1$ between node $q = 2$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{22} = z_{20} = j0.10$$

Next, add branch 2, $z_{30} = j0.1$ between node $q = 3$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{22} & 0 \\ 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.1$ between the new node $q = 1$ and the existing node $p = 3$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{33} + z_{13} & 0 & Z_{33} \\ 0 & Z_{22} & 0 \\ Z_{33} & 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix}$$

Add link 4, $z_{12} = j0.2$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} j0.2 & 0 & j0.1 & -j0.2 \\ 0 & j0.1 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & -j0.1 \\ -j0.2 & j0.1 & -j0.1 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.2 + j0.2 + j0.1 - 2(j0) = j0.5$$

and

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j0.5}\begin{bmatrix} -j0.2 \\ j0.1 \\ -j0.1 \end{bmatrix} \begin{bmatrix} -j0.2 & j0.1 & -j0.1 \end{bmatrix}$$

$$= \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix} - \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix}$$

$$= \begin{bmatrix} j0.12 & j0.04 & j0.06 \\ j0.04 & j0.08 & j0.02 \\ j0.06 & j0.02 & j0.08 \end{bmatrix}$$

9.8. Obtain the bus impedance matrix for the network of Problem 9.4.

Add branch 1, $z_{10} = j0.05$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.05$$

Next, add branch 2, $z_{20} = j0.075$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 \\ 0 & j0.075 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.3$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.75$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$
$$= \begin{bmatrix} j0.05 & 0 & j0.05 & -j0.05 \\ 0 & j0.075 & 0 & j0.075 \\ j0.05 & 0 & j0.35 & -j0.05 \\ -j0.05 & j0.075 & -j0.05 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.75 + j0.05 + j0.075 - 2(j0) = j0.875$$

and

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j0.875} \begin{bmatrix} -j0.05 \\ j0.075 \\ -j0.05 \end{bmatrix} \begin{bmatrix} -j0.05 & j0.075 & -j0.05 \end{bmatrix}$$
$$= \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix}$$
$$= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix}$$

Add link 5, $z_{23} = j0.45$ between node $q = 3$ and node $p = 2$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(5)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13} - Z_{12} \\ Z_{21} & Z_{22} & Z_{23} & Z_{23} - Z_{22} \\ Z_{31} & Z_{32} & Z_{33} & Z_{33} - Z_{32} \\ Z_{31} - Z_{21} & Z_{32} - Z_{22} & Z_{33} - Z_{23} & Z_{44} \end{bmatrix}$$
$$= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 & j0.042857 \\ j0.004286 & j0.068571 & j0.004286 & -j0.064286 \\ j0.047143 & j0.004286 & j0.347142 & j0.342857 \\ j0.042857 & -j0.064286 & j0.342857 & Z_{44} \end{bmatrix}$$

From (9.58)

$$\begin{aligned} Z_{44} &= z_{23} + Z_{22} + Z_{33} - 2Z_{23} = j0.45 + j0.068571 + j0.347142 - \\ &\quad 2(j0.004286) = j0.85714 \end{aligned}$$

and

$$\begin{aligned} \frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{j0.85714}\begin{bmatrix} j0.042857 \\ -j0.064286 \\ j0.342857 \end{bmatrix}\begin{bmatrix} j0.042857 & -j0.064286 & j0.342857 \end{bmatrix} \\ &= \begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137143 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix} -$$

$$\begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137142 \end{bmatrix} = \begin{bmatrix} j0.0450 & j0.00750 & j0.030 \\ j0.0075 & j0.06375 & j0.030 \\ j0.0300 & j0.03000 & j0.210 \end{bmatrix}$$

9.9. The bus impedance matrix for the network shown in Figure 77 is given by

$$Z_{bus} = j\begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix}$$

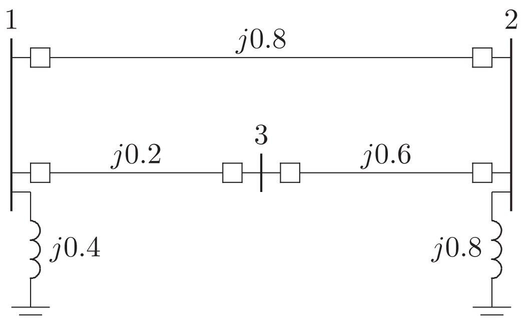


FIGURE 77
One-line diagram for Problem 9.9.

There is a line outage and the line from bus 1 to 2 is removed. Using the method of building algorithm determine the new bus impedance matrix.

The line between buses 1 and 2 with impedance $Z_{12} = j0.8$ is removed. The removal of this line is equivalent to connecting a link having an impedance equal to the negated value of the original impedance. Therefore, we add link $z_{12} = -j0.8$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

Thus, we get

$$\mathbf{Z}_{bus}^{(1)} = \begin{bmatrix} j0.300 & j0.200 & j0.27500 & -j0.100 \\ j0.200 & j0.400 & j0.25000 & j0.200 \\ j0.275 & j0.250 & j0.41875 & -j0.025 \\ -j0.100 & j0.200 & -j0.02500 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = -j0.8 + j0.3 + j0.4 - 2(j0.2) = -j0.5$$

and

$$\begin{aligned} \frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{-j0.5} \begin{bmatrix} -j0.100 \\ j0.200 \\ -j0.025 \end{bmatrix} \begin{bmatrix} -j0.10 & j0.20 & -j0.025 \end{bmatrix} \\ &= \begin{bmatrix} -j0.020 & j0.040 & -j0.0050 \\ j0.040 & -j0.080 & j0.0100 \\ -j0.005 & j0.010 & -j0.0013 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus} &= \begin{bmatrix} j0.300 & j0.200 & j0.27500 \\ j0.200 & j0.400 & j0.25000 \\ j0.275 & j0.250 & j0.41875 \end{bmatrix} - \begin{bmatrix} -j0.020 & j0.040 & -j0.00500 \\ j0.040 & -j0.080 & j0.01000 \\ -j0.005 & j0.010 & -j0.00125 \end{bmatrix} \\ &= \begin{bmatrix} j0.320 & j0.160 & j0.280 \\ j0.160 & j0.480 & j0.240 \\ j0.280 & j0.240 & j0.420 \end{bmatrix} \end{aligned}$$

9.10. The per unit bus impedance matrix for the power system of Problem 9.4 is given by

$$Z_{bus} = j \begin{bmatrix} 0.0450 & 0.0075 & 0.0300 \\ 0.0075 & 0.06375 & 0.0300 \\ 0.0300 & 0.0300 & 0.2100 \end{bmatrix}$$

A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a fault at bus 3 with fault impedance $Z_f = j0.19$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \ \text{pu}$$
$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \ \text{pu}$$
$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.21)(-j2.5) = 0.475 \ \text{pu}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \ \text{pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \ \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \ \text{pu} \end{aligned}$$

To check the result, we form the line data for the network of Problem 9.4, and we use the following commands

```
zdata=[0   1    0    0.05
       0   2    0    0.075
       1   2    0    0.75
       1   3    0    0.30
       2   3    0    0.45];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

```
Zbus =

     0 + 0.0450i     0 + 0.0075i     0 + 0.0300i
     0 + 0.0075i     0 + 0.0638i     0 + 0.0300i
     0 + 0.0300i     0 + 0.0300i     0 + 0.2100i
```

```
Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*.19

Balanced three-phase fault at bus No. 3
Total fault current =   2.5000 per unit

Bus Voltages during fault in per unit

     Bus     Voltage       Angle
     No.     Magnitude     degrees
      1       0.9250       0.0000
      2       0.9250       0.0000
      3       0.4750       0.0000

Line currents for fault at bus No.  3

     From      To     Current     Angle
     Bus       Bus    Magnitude   degrees
      G         1      1.5000    -90.0000
      1         2      0.0000    -90.0000
      1         3      1.5000    -90.0000
      G         2      1.0000    -90.0000
      2         3      1.0000    -90.0000
      3         F      2.5000    -90.0000
Another fault location? Enter'y' or'n' within single quote->'n'
```

9.11. The per unit bus impedance matrix for the power system of Problem 9.5 is given by

$$Z_{bus} = j \begin{bmatrix} 0.240 & 0.140 & 0.200 & 0.200 \\ 0.140 & 0.2275 & 0.175 & 0.175 \\ 0.200 & 0.175 & 0.310 & 0.310 \\ 0.200 & 0.1750 & 0.310 & 0.500 \end{bmatrix}$$

(a) A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.
(b) Repeat (a) for a three-phase fault at bus 2 with a fault impedance of $Z_f = j0.0225$.
(c) Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a solid fault at bus 4 the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44}} = \frac{1.0}{j0.5} = -j2 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned}
V_1(F) &= V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.200)(-j2) = 0.60 \text{ pu}\\
V_2(F) &= V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.175)(-j2) = 0.65 \text{ pu}\\
V_3(F) &= V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.310)(-j2) = 0.38 \text{ pu}\\
V_4(F) &= V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.500)(-j2) = 0 \text{ pu}
\end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned}
I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.60}{j0.5} = -j0.1 \text{ pu}\\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = -j1.1 \text{ pu}\\
I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = -j0.9 \text{ pu}\\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = -j2 \text{ pu}
\end{aligned}$$

(c) From (9.22), for a fault at bus 2 with fault impedance $Z_f = j0.0225$ per unit, the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned}
V_1(F) &= V_1(0) - Z_{12}I_2(F) = 1.0 - (j0.140)(-j4) = 0.44 \text{ pu}\\
V_2(F) &= V_2(0) - Z_{22}I_2(F) = 1.0 - (j0.2275)(-j4) = 0.09 \text{ pu}\\
V_3(F) &= V_3(0) - Z_{32}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \text{ pu}\\
V_4(F) &= V_4(0) - Z_{42}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \text{ pu}
\end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned}
I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = -j0.7 \text{ pu}\\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = -j0.7 \text{ pu}\\
I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = -j0.7 \text{ pu}\\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.30}{j0.19} = 0 \text{ pu}
\end{aligned}$$

To check the result, we form the line data for the network of Problem 9.11, and we use the following commands

```
zdata=[0   1    0   0.40
       0   2    0   0.35
       1   2    0   0.50
       1   3    0   0.20
       2   3    0   0.30
       3   4    0   0.19];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

```
Zbus =
      0 + 0.2400i   0 + 0.1400i   0 + 0.2000i   0 + 0.2000i
      0 + 0.1400i   0 + 0.2275i   0 + 0.1750i   0 + 0.1750i
      0 + 0.2000i   0 + 0.1750i   0 + 0.3100i   0 + 0.3100i
      0 + 0.2000i   0 + 0.1750i   0 + 0.3100i   0 + 0.5000i

Enter Faulted Bus No. -> 4

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Balanced three-phase fault at bus No. 4
Total fault current =   2.0000 per unit

Bus Voltages during fault in per unit

     Bus     Voltage       Angle
     No.     Magnitude     degrees
      1       0.6000       0.0000
      2       0.6500       0.0000
      3       0.3800       0.0000
      4       0.0000       0.0000

Line currents for fault at bus No.  4

     From       To     Current     Angle
     Bus        Bus    Magnitude   degrees
      G          1      1.0000    -90.0000
      1          3      1.1000    -90.0000
      G          2      1.0000    -90.0000
      2          1      0.1000    -90.0000
      2          3      0.9000    -90.0000
      3          4      2.0000    -90.0000
      4          F      2.0000    -90.0000
```

```
Another fault location? Enter'y' or'n' within single quote->'y'
Enter Faulted Bus No. -> 2

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.0225
Balanced three-phase fault at bus No. 2
Total fault current =   4.0000 per unit

Bus Voltages during fault in per unit

     Bus      Voltage        Angle
     No.      Magnitude      degrees
      1        0.4400        0.0000
      2        0.0900        0.0000
      3        0.3000        0.0000
      4        0.3000        0.0000

Line currents for fault at bus No.  2

     From       To     Current     Angle
     Bus        Bus    Magnitude   degrees
      G          1      1.4000    -90.0000
      1          2      0.7000    -90.0000
      1          3      0.7000    -90.0000
      G          2      2.6000    -90.0000
      2          F      4.0000    -90.0000
      3          2      0.7000    -90.0000
```

9.12. The per unit bus impedance matrix for the power system shown in Figure 78 is given by

$$Z_{bus} = j\begin{bmatrix} 0.150 & 0.075 & 0.140 & 0.135 \\ 0.075 & 0.1875 & 0.090 & 0.0975 \\ 0.140 & 0.090 & 0.2533 & 0.210 \\ 0.135 & 0.0975 & 0.210 & 0.2475 \end{bmatrix}$$

A three-phase fault occurs at bus4 through a fault impedance of $Z_f = j0.0025$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a fault at bus 2 with fault impedance $Z_f = j0.0225$ per unit, the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44} + Z_f} = \frac{1.0}{j0.2475 + j0.0025} = -j4 \quad \text{pu}$$

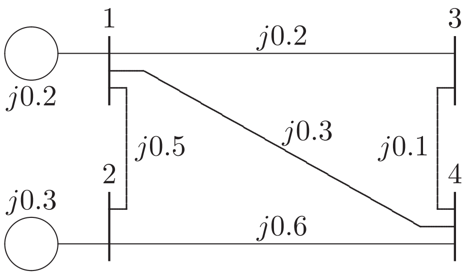


FIGURE 78
One-line diagram for Problem 9.12.

From (9.23), bus voltages during the fault are

$$\begin{aligned}
V_1(F) &= V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.135)(-j4) = 0.46 \text{ pu} \\
V_2(F) &= V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.0975)(-j4) = 0.61 \text{ pu} \\
V_3(F) &= V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.210)(-j4) = 0.16 \text{ pu} \\
V_4(F) &= V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.2475)(-j4) = 0.01 \text{ pu}
\end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned}
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.46 - 0.16}{j0.2} = -j1.5 \text{ pu} \\
I_{14}(F) &= \frac{V_1(F) - V_4(F)}{z_{14}} = \frac{0.46 - 0.01}{j0.3} = -j1.5 \text{ pu} \\
I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.61 - 0.46}{j0.5} = -j0.3 \text{ pu} \\
I_{24}(F) &= \frac{V_2(F) - V_4(F)}{z_{24}} = \frac{0.61 - 0.01}{j0.6} = -j1.0 \text{ pu} \\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.16 - 0.01}{j0.10} = -j1.5 \text{ pu}
\end{aligned}$$

To check the result, we form the line data for the network of Problem 9.12, and we use the following commands

```
zdata=[0   1    0   0.2
       0   2    0   0.3
       1   2    0   0.5
       1   3    0   0.2
       1   4    0   0.3
       2   4    0   0.6
       3   4    0   0.1];
```

```
    Zbus=zbuild(zdata)
    symfault(zdata, Zbus)
```

The result is

```
Zbus =
      0 + 0.1500i    0 + 0.0750i    0 + 0.1400i    0 + 0.1350i
      0 + 0.0750i    0 + 0.1875i    0 + 0.0900i    0 + 0.0975i
      0 + 0.1400i    0 + 0.0900i    0 + 0.2533i    0 + 0.2100i
      0 + 0.1350i    0 + 0.0975i    0 + 0.2100i    0 + 0.2475i

Enter Faulted Bus No. -> 4

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.0025

Balanced three-phase fault at bus No. 4
Total fault current =   4.0000 per unit

Bus Voltages during fault in per unit

     Bus      Voltage       Angle
     No.      Magnitude     degrees
      1        0.4600       0.0000
      2        0.6100       0.0000
      3        0.1600       0.0000
      4        0.0100       0.0000

Line currents for fault at bus No.  4

     From      To     Current     Angle
     Bus       Bus    Magnitude   degrees
      G         1      2.7000    -90.0000
      1         3      1.5000    -90.0000
      1         4      1.5000    -90.0000
      G         2      1.3000    -90.0000
      2         1      0.3000    -90.0000
      2         4      1.0000    -90.0000
      3         4      1.5000    -90.0000
      4         F      4.0000    -90.0000
Another fault location? Enter'y' or'n' within single quote->'n'
```

9.13. Repeat Example 9.7 for a bolted three-phase fault at bus 9.

Run chp9ex7 for for a bolted three-phase fault at bus 9 **ch9p13**.

9.14. Repeat Example 9.8 for a bolted three-phase fault at bus 9.

Run chp9ex8 for for a bolted three-phase fault at bus 9 **ch9p14**.

9.15. Repeat Example 9.9 for a bolted three-phase fault at bus 9.

Run chp9ex9 for a bolted three-phase fault at bus 9. The impedances are expressed on a 1000 MVA base. In Example 9.9 the base is mistakenly stated as 100 MVA **ch9p15**.

9.16. The 6-bus power system network of an electric utility company is shown in Figure 79. The line and transformer data containing the series resistance and reac-

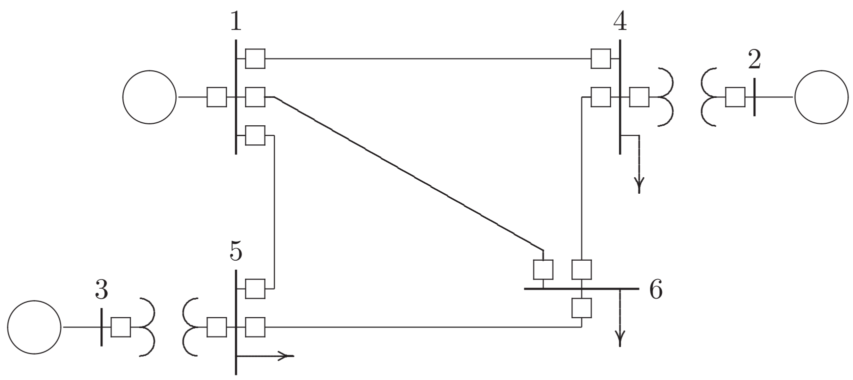


FIGURE 79
One-line diagram for Problem 9.16.

tance in per unit, and one-half of the total capacitance in per unit susceptance on a 100-MVA base, is tabulated below.

LINE AND TRANSFORMER DATA				
Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}$ B, PU
1	4	0.035	0.225	0.0065
1	5	0.025	0.105	0.0045
1	6	0.040	0.215	0.0055
2	4	0.000	0.035	0.0000
3	5	0.000	0.042	0.0000
4	6	0.028	0.125	0.0035
5	6	0.026	0.175	0.0300

The transient impedance of the generators on a 100-MVA base are given below.

GEN. TRANSIENT IMPEDANCE, PU		
Gen. No.	R_a	X'_d
1	0	0.20
2	0	0.15
3	0	0.25

Neglecting the shunt capacitors and the loads, use **Zbus = zbuild(zdata)** function to obtain the bus impedance matrix. Assuming all the prefault bus voltages are equal to $1\angle 0°$, use **symfault(zdata, Zbus)** function to compute the fault current, bus voltages, and line currents for a bolted fault at bus 6. When using **Zbus = zbuild(zdata)** function, the generator reactances must be included in the **zdata** array with bus zero as the reference bus.

Use the following data, and run to obtain the result.

```
zdata = [ 1     0     0.000   0.200
          2     0     0.000   0.150
          3     0     0.000   0.250
          1     4     0.035   0.225
          1     5     0.025   0.105
          1     6     0.040   0.215
          2     4     0.000   0.035
          3     5     0.000   0.042
          4     6     0.028   0.125
          5     6     0.026   0.175];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

9.17. In Problem 9.16 consider the shunt capacitors and neglect the loads. use **zbuildpi(linedata, gendata, yload)** function to obtain the bus impedance matrix. Assuming all the prefault bus voltages are equal to $1\angle 0°$, use **symfault(linedata, Zbus)** function to compute the fault current, bus voltages, and line currents for a bolted fault at bus 6.

Use the following data, and run to obtain the result.

```
linedata = [ 1     4     0.035   0.225   0.0065
             1     5     0.025   0.105   0.0045
             1     6     0.040   0.215   0.0055
             2     4     0.000   0.035   0.0000
             3     5     0.000   0.042   0.0000
             4     6     0.028   0.125   0.0035
```

```
             5     6     0.026   0.175   0.0300];
%           Gen. Ra     Xd'
gendata = [ 1    0     0.20
            2    0     0.15
            3    0     0.25];
Zbus=zbuildpi(linedata, gendata)
symfault(linedata, Zbus)
```

9.18. Repeat the symmetrical three-phase short circuit analysis for Problem 9.16 considering the prefault bus voltages and the effect of load currents. The load data is as follows.

LOAD DATA		
Bus	Load	
No.	MW	Mvar
1	0	0
2	0	0
3	0	0
4	100	70
5	90	30
6	160	110

Voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.06\angle 0°$, is taken as the slack bus.

GENERATION DATA				
Bus	Voltage	Generation,	Mvar Limits	
No.	Mag.	MW	Min.	Max.
1	1.060			
2	1.040	150.0	0.0	140.0
3	1.030	100.0	0.0	90.0

Use anyone of the power flow programs to obtain the prefault bus voltages and the load admittance. The power flow program returns the prefault bus voltage array **V** and the bus load admittance array **yload**.

Use the following data, and run to obtain the result.

```
basemva = 100;  accuracy = 0.0001;  maxiter = 10;
%      Bus Bus  |V|  Ang --Load--   --Gen--           Injected
%      No. code  pu  Deg MW   Mvar   MW  Mvar Min  Max   Mvar
busdata=[1   1  1.06  0   00.0   0.0  0.0  0.0  0    0    0
         2   2  1.04  0  150.0   0.0  0.0  0.0  0  140    0
```

```
         3   2  1.03  0  100.0   0.0  0.0  0.0  0   90    0
         4   0  1.0   0  100.0  70.0  0.0  0.0  0    0    0
         5   0  1.0   0   90.0  30.0  0.0  0.0   0   0    0
         6   0  1.0   0  160.0 110.0  0.0  0.0   0   0    0];
%            Bus   Bus     R       X       1/2B
%            No.   No.     pu      pu    pu Line code
linedata = [ 1     4     0.035   0.225   0.0065   1
             1     5     0.025   0.105   0.0045   1
             1     6     0.040   0.215   0.0055   1
             2     4     0.000   0.035   0.0000   1
             3     5     0.000   0.042   0.0000   1
             4     6     0.028   0.125   0.0035   1
             5     6     0.026   0.175   0.0300   1];
%         Gen. Ra    Xd'
gendata=[ 1    0    0.20
          2    0    0.15
          3    0    0.25];
lfybus                       % Forms the bus admittance matrix
lfnewton        % Power flow solution by Newton-Raphson method
busout          % Prints the power flow solution on the screen
Zbus=zbuildpi(linedata, gendata, yload)
                                %Forms Zbus including the load
symfault(linedata,Zbus,V)%3-phase fault including load current
```

CHAPTER 10 PROBLEMS

10.1. Obtain the symmetrical components for the set of unbalanced voltages $V_a = 300\angle{-120°}$, $V_b = 200\angle 90°$, and $V_c = 100\angle{-30°}$.
The commands

```
Vabc = [300  -120
        200    90
        100   -30];
V012 = abc2sc(Vabc);  % Symmetrical components of phase a
V012p= rec2pol(V012)          % Rectangular to polar form
```

result in

```
V012p =
         42.2650 -120.0000
        193.1852 -135.0000
          86.9473  -84.8961
```

10.2. The symmetrical components of a set of unbalanced three-phase currents are $I_a^0 = 3\angle{-30°}$, $I_a^1 = 5\angle 90°$, and $I_a^2 = 4\angle 30°$. Obtain the original unbalanced phasors.

The commands

```
I012 = [3    -30
        5     90
        4     30];
Iabc = sc2abc(I012);%Unbalanced phasor to symmetrical comp.
Iabcp= rec2pol(Iabc)           % Rectangular to polar form
```

result in

```
Iabcp =
         8.1854   42.2163
         4.0000  -30.0000
         8.1854 -102.2163
```

10.3. The operator a is defined as $a = 1\angle 120°$; show that
(a) $\frac{(1+a)}{(1+a^2)} = 1\angle 120°$
(b) $\frac{(1-a)^2}{(1+a)^2} = 3\angle -180°$
(c) $(a - a^2)(a^2 - a) = 3\angle 0°$
(d) $V_{an}^1 = \frac{1}{\sqrt{3}} V_{bc}^1 \angle 90°$
(e) $V_{an}^2 = \frac{1}{\sqrt{3}} V_{bc}^2 \angle -90°$

(a) Since $1 + a + a^2 = 0$, we have

$$\frac{(1+a)}{(1+a^2)} = \frac{1+a}{-a} = -\frac{1}{a} - 1$$
$$= 0.5 + j0.866 - 1 = 1\angle 120°$$

(b)

$$\frac{(1-a)^2}{(1+a)^2} = \frac{(1-a)^2}{(-a^2)^2} = \frac{1-2a+a^2}{a} = \frac{1}{a} - 2 + a =$$
$$= -0.5 - j0.866 - 2 - 0.5 + j0.866 = 3\angle 180°$$

(c)

$$(a - a^2)(a^2 - a) = 2a^3 - a^2 - a^4 = 2a^3 - (a^2 + a)$$
$$= 2(1) - (-1) = 3$$

(d)

$$V_{bc}^1 = V_{bn}^1 - V_{cn}^1 = a^2 V_{an}^1 - a V_{an}^1 = (a^2 - a) V_{an}^1$$
$$= (-0.5 - j0.866 + 0.5 - j0.866) V_{an}^1$$
$$= \sqrt{3}\angle -90° V_{an}^1$$

or

$$V_{an}^1 = \frac{1}{\sqrt{3}} V_{bc}^1 \angle 90°$$

(e)

$$V_{bc}^2 = V_{bn}^2 - V_{cn}^2 = aV_{an}^2 - a^2V_{an}^2 = (a - a^2)V_{an}^2$$
$$= (-0.5 + j0.866 + 0.5 + j0.866)V_{an}^2$$
$$= \sqrt{3}\angle 90^\circ V_{an}^2$$

or

$$V_{an}^2 = \frac{1}{\sqrt{3}} V_{bc}^2 \angle -90^\circ$$

10.4. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 1000\angle 0^\circ$, $V_{bc} = 866.0254\angle -150^\circ$, and $V_{ca} = 500\angle 120^\circ$. Determine the symmetrical components for line and phase voltages, then find the phase voltages V_{an}, V_{bn}, and V_{cn}.

First find the symmetrical components of line voltages, then find the symmetrical components of phase voltages. Use the inverse symmetrical components transformation to obtain the phase voltages. We use the following commands

```
a = -0.5+j*sqrt(3)/2;
Vabbcca=[1000      0        % Unbalanced line-to-line voltage
         866.0254  -150
         500        120];
VL012=abc2sc(Vabbcca); % Sym. comp. line voltages, rectangular
VL012p=rec2pol(VL012)        % Sym. comp. line voltages, polar
Va012=[ 0              % Sym. comp. phase voltages, rectangular
        VL012(2)/(sqrt(3)*(0.866+j0.5))
        VL012(3)/(sqrt(3)*(0.866-j0.5))];
Va012p=rec2pol(Va012)        % Sym. comp. phase voltage, polar
Vabc=sc2abc(Va012);    % Unbalanced phase voltages, rectangular
Vabcp=rec2pol(Vabc)         % Unbalanced phase voltages, polar
```

The result is

```
VL012p =
           0.0000   30.0000
         763.7626  -10.8934
         288.6751   30.0000

Va012p =
           0         0
         440.9586  -40.8934
         166.6667   60.0000
```

```
Vabcp =
          440.9586  -19.1066
          600.9252 -166.1021
          333.3333   60.0000
```

10.5. In the three-phase system shown in Figure 80, phase a is on no load and phases b and c are short-circuited to ground.

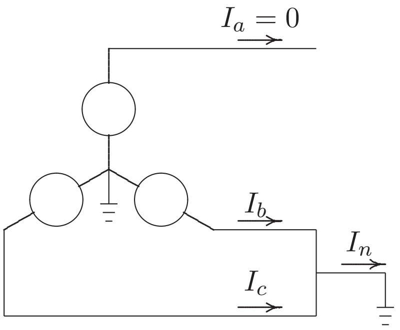


FIGURE 80
Circuit for Problem 10.5.

The following currents are given:

$I_b = 91.65\angle 160.9^\circ$

$I_n = 60.00\angle 90^\circ$

Find the symmetrical components of current I_a^0, I_a^1, and I_a^2.

$$I_c = I_n - I_b = 60\angle 90^\circ - 91.65\angle 160.9^\circ = 91.6569\angle 19.1124$$

We use the following commands

```
Iabc = [  0         0
         91.65    160.9
         91.6569   19.1124];
I012=abc2sc(Iabc);  % Sym. components of currents, rectangular
I012p = rec2pol(I012)    % Sym. components of currents , polar
```

The result is

```
I012p =
          20.0000   90.0000
          60.0012  -89.9942
          40.0012   90.0087
```

10.6. A balanced three-phase voltage of 360-V line-to-neutral is applied to a balanced Y-connected load with ungrounded neutral, as shown in Figure 81. The three-phase load consists of three mutually-coupled reactances. Each phase has a series reactance of $Z_s = j24\ \Omega$, and the mutual coupling between phases is $Z_m = j6\ \Omega$.
(a) Determine the line currents by mesh analysis without using symmetrical components.
(b) Determine the line currents using symmetrical components.

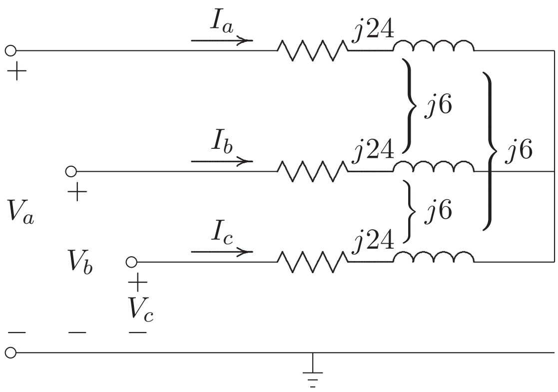


FIGURE 81
Circuit for Problem 10.6.

(a) Applying KVL to the two independent mesh equations, and writing one node equation, results in

$$\begin{bmatrix} (Z_s - Z_m) & -(Z_s - Z_m) & 0 \\ 0 & (Z_s - Z_m) & -(Z_s - Z_m) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} |V_L|\angle \pi/6 \\ |V_L|\angle -\pi/2 \\ 0 \end{bmatrix}$$

or in compact form

$$\mathbf{Z}_{mesh}\mathbf{I}^{abc} = \mathbf{V}_{mesh}$$

Solving the above equations results in the line currents

$$\mathbf{I}^{abc} = \mathbf{Z}_{mesh}{}^{-1}\mathbf{V}_{mesh}$$

The following commands

```
disp('(a) Solution by mesh analysis')
VL=360*sqrt(3);                          % Line voltage magnitude
Zs=j*24; Zm=j*6;                     % Series and mutual impedances
Z= [(Zs-Zm)  -(Zs-Zm)      0
```

```
       0       (Zs-Zm)  -(Zs-Zm) % Matrix from 2 mesh equations
       1          1         1   ];      % and one node equation
V=[VL*cos(pi/6)+j*VL*sin(pi/6)    % RHS of mesh node equations
   VL*cos(-pi/2)+j*VL*sin(-pi/2)
             0                  ];
Iabc=Z\V;                         % Line currents, Rectangular
Iabcp=[abs(Iabc), angle(Iabc)*180/pi]   % Line currents, polar
```

The result is

```
(a) Solution by mesh analysis
    Iabcp =
           20.0000  -90.0000
           20.0000  150.0000
           20.0000   30.0000
```

(b) Using the symmetrical components method, we have

$$\mathbf{V}^{012} = \mathbf{Z}^{012}\mathbf{I}^{012}$$

where

$$\mathbf{V}^{012} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

and from (10.32)

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

for the sequence components of currents, we get

$$\mathbf{I}^{012} = [\mathbf{Z}^{012}]^{-1}\mathbf{V}^{012}$$

We write the following commands

```
disp('Solution by Symmetrical components method' )
Z012=[Zs+2*Zm   0     0        % Symmetrical components matrix
        0     Zs-Zm   0
        0       0   Zs-Zm];
V012=[0; VL/sqrt(3); 0];%Symmetrical components phase voltages
I012=inv(Z012)*V012  % Symmetrical components of line currents
a=cos(2*pi/3)+j*sin(2*pi/3);
A=[ 1  1  1; 1 a^2  a; 1 a a^2];       % Transformation matrix
Iabc=A*I012;                      % Line currents, rectangular
Iabcp=[abs(Iabc), angle(Iabc)*180/pi]   % Line currents, Polar
```

which result in

```
(b) Solution by Symmetrical components method
   I012 =
          0
          0 -20.0000i
          0
   Iabcp =
          20.0000  -90.0000
          20.0000  150.0000
          20.0000   30.0000
```

This is the same result as in part (a).

10.7. A three-phase unbalanced source with the following phase-to-neutral voltages

$$\mathbf{V}^{abc} = \begin{bmatrix} 300 & \angle -120^\circ \\ 200 & \angle 90^\circ \\ 100 & \angle -30^\circ \end{bmatrix}$$

is applied to the circuit in Figure 82. The load series impedance per phase is

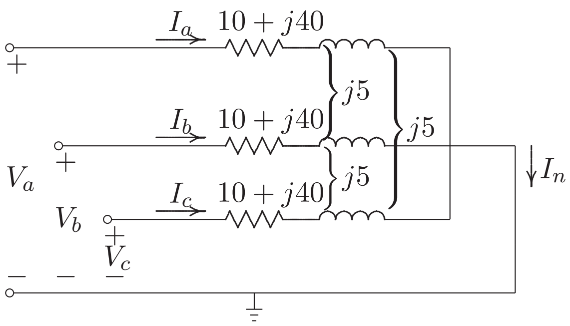


FIGURE 82
Circuit for Problem 10.7.

$Z_s = 10 + j40$ and the mutual impedance between phases is $Z_m = j5$. The load and source neutrals are solidly grounded. Determine
(a) The load sequence impedance matrix, $\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}$.
(b) The symmetrical components of voltage.
(c) The symmetrical components of current.
(d) The load phase currents.
(e) The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_a^0 I_a^{0*} + V_a^1 I_a^{1*} + V_a^2 I_a^{2*})$.

(f) The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.

We write the following commands

```
Vabc=[300    -120                   % Phase-to-neutral voltages
      200      90
      100     -30];
Zabc=[10+j*40    j*5    j*5 %Self and mutual impedances matrix
         j*5  10+j*40   j*5
         j*5     j*5 10+j*40];
Z012 = zabc2sc(Zabc)     % Symmetrical components of impedance
V012 = abc2sc(Vabc);       % Symmetrical components of voltage
V012p= rec2pol(V012)   % Converts rectangular phasors to polar
I012 = inv(Z012)*V012 ;    % Symmetrical components of current
I012p= rec2pol(I012)   % Converts rectangular phasors to polar
Iabc = sc2abc(I012);                              % Phase currents
Iabcp= rec2pol(Iabc)   % Converts rectangular phasors to polar
S3ph=3*(V012.')*conj(I012) %Power using symmetrical components
Vabcr = Vabc(:,1).*(cos(pi/180*Vabc(:,2))+...
j*sin(pi/180*Vabc(:,2)));
S3ph=(Vabcr.')*conj(Iabc)       % Power using phase quantities
```

The result is

```
Z012 =
       10.0+50.0i    0              0
        0           10.0 +35.0i     0
        0            0             10.0+35.0i
V012p =
        42.2650   -120.0000
       193.1852   -135.0000
        86.9473    -84.8961

I012p =
         0.8289    161.3099
         5.3072    150.9454
         2.3886   -158.9507
Iabcp =
         7.9070    165.4600
         5.8190     14.8676
         2.7011    -96.9315
S3ph =
         1036.8+3659.6i
S3ph =
         1036.8+3659.6i
```

10.8. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 600\angle 36.87°$, $V_{bc} = 800\angle 126.87°$, and $V_{ca} = 1000\angle -90°$. A Y-connected load with a resistance of $37\ \Omega$ per phase is connected to the supply. Determine
(a) The symmetrical components of voltage.
(b) The phase voltages.
(c) The line currents.

We use the following statements

```
Vabbcca=[600  36.87                 % Unbalanced line voltages
         800  126.87
         1000  -90];
VL012=abc2sc(Vabbcca); % Sym. comp. line voltages, rectangular
VL012p=rec2pol(VL012)        % Sym. comp. line voltages, polar
Va012=[0
       VL012(2)/(sqrt(3)*(0.866+j*.5))
       VL012(3)/(sqrt(3)*(0.866-j*.5))];  % Sym. components of
                                 % phase voltages, rectangular
Va012p=rec2pol(Va012)    % Sym. comp. of phase voltages, polar
Vabc=sc2abc(Va012);              % Phase voltages, rectangular
Vabcp=rec2pol(Vabc)                    % Phase voltages, polar
Iabc=Vabc/37;                     % Line currents, rectangular
Iabcp=rec2pol(Iabc)                     % Line currents, polar
```

which result in

```
   VL012p =
            0.0006   -179.9999
          237.0762    169.9342
          781.3204     24.0621

   Va012p =
            0         0
          136.8790    139.9335
          451.1055     54.0628

   Vabcp =
          480.7542     70.5606
          333.3386    163.7411
          569.6111    -73.6857

   Iabcp =
           12.9934     70.5606
            9.0092    163.7411
           15.3949    -73.6857
```

10.9. A generator having a solidly grounded neutral and rated 50-MVA, 30-kV has positive-, negative-, and zero-sequence reactances of 25, 15, and 5 percent, respectively. What reactance must be placed in the generator neutral to limit the fault current for a bolted line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \ \ \Omega$$

The three-phase fault current is

$$I_{f\,3\phi} = \frac{1}{0.25} = 4.0 \ \ \text{pu}$$

The line-to-ground fault current is

$$I_{f\,LG} = \frac{3}{0.25 + 0.15 + 0.05 + 3X_n} = 4.0 \ \ \text{pu}$$

Solving for X_n, results in

$$\begin{aligned} X_n &= 0.1 \quad \text{pu} \\ &= (0.1)(18) = 1.8 \ \ \Omega \end{aligned}$$

10.10. What reactance must be placed in the neutral of the generator of Problem 9 to limit the magnitude of the fault current for a bolted double line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \ \ \Omega$$

From (10.86) the positive sequence component of fault current is

$$I_a^1 = \frac{1}{0.25 + \frac{(0.15)(0.05+3X_n)}{0.2+3X_n}} = \frac{0.2 + 3X_n}{0.0575 + 1.2X_n}$$

and from (10.86) the zero sequence component of fault current is

$$I_a^0 = -\frac{1 - \frac{(0.25)(0.2+3X_n)}{0.0575+1.2X_n}}{0.05 + 3X_n} = -\frac{0.0075 + 0.45X}{3.6X^2 + 0.2325X + 0.002875}$$

The double line-to-ground fault current is

$$I_{f\,DLG} = 3I_a^0$$

Since the magnitude of the fault current is to be equal to 4, we get

$$|I_a^0| = \frac{4}{3}$$

Substituting for I_a^0, we have

$$\frac{4}{3} = \frac{0.0075 + 0.45X}{3.6X^2 + 0.2325X + 0.002875}$$

which results in the following second-order polynomial equation

$$14.4x^2 - 0.42X - 0.011 = 0$$

The positive root of the above polynomial is $X = 0.0458333$ or

$$X_n = (0.0458333)(18) = 0.825 \ \ \Omega$$

10.11. Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown in Figure 83. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 Ω. The generator data and the reactance of the reactors are tabulated below. A line-to-ground fault occurs on phase a of the common bus bar. Neglect prefault currents and assume generators are operating at their rated voltage. Determine the fault current in phase a.

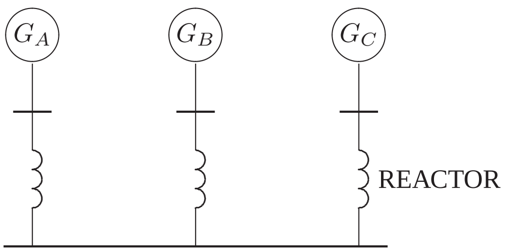


FIGURE 83
Circuit for Problem 10.11.

Item	X^1	X^2	X^0
G_A	0.25 pu	0.155 pu	0.056 pu
G_B	0.20 pu	0.155 pu	0.056 pu
G_C	0.20 pu	0.155 pu	0.060 pu
Reactor	6.0 Ω	6.0 Ω	6.0 Ω

The generator base impedance is

$$Z_B = \frac{(30)^2}{15} = 60 \ \ \Omega$$

The reactor per-unit reactance, and the per-unit generator C neutral reactor are

$$X_R = \frac{6}{60} = 0.1 \text{ pu}$$
$$X_n = \frac{2}{60} = 0.3333 \text{ pu}$$

The positive-sequence impedance network is shown in Figure 84(a), and the zero-sequence impedance network is shown in Figure 84(b).

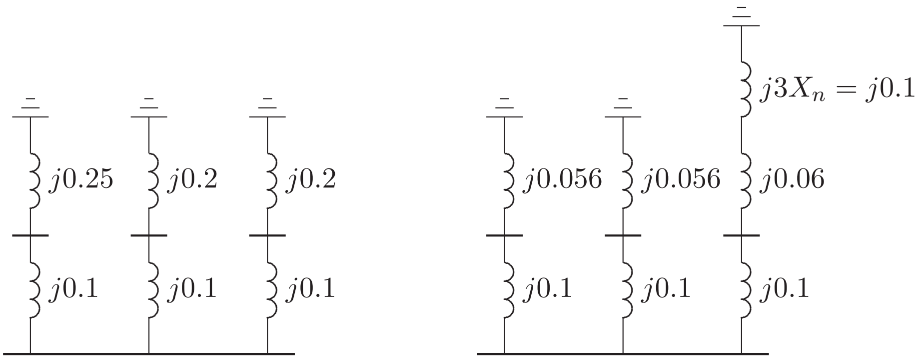


FIGURE 84
Positive- and zero-sequence impedance networks for Problem 10.11.

The positive-sequence impedance is

$$\frac{1}{X^1} = \frac{1}{0.35} + \frac{1}{0.3} + \frac{1}{0.3} \quad \text{or} \quad X^1 = 0.105$$

The negative-sequence impedance network is the same as the positive-sequence impedance network, except for the value of the generator negative-sequence reactance. Therefore, the negative-sequence impedance is

$$\frac{1}{X^2} = \frac{1}{0.255} + \frac{1}{0.255} + \frac{1}{0.255} \quad \text{or} \quad X^2 = 0.085$$

The zero-sequence impedance is

$$\frac{1}{X^0} = \frac{1}{0.156} + \frac{1}{0.156} + \frac{1}{0.26} \quad \text{or} \quad X^0 = 0.06$$

The line-to-ground fault current in phase a is

$$I_a = 3I_a^0 = \frac{3(1)}{j(0.105 + 0.085 + 0.06)} = 12\angle -90^\circ \text{ pu}$$

10.12. Repeat Problem 10.11 for a bolted line-to-line fault between phases b and c.

The positive-sequence fault current in phase a is

$$I_a^1 = \frac{1}{Z^1 + Z^2} = \frac{1}{j(0.105 + 0.085)} = -j5.26316 \ \text{pu}$$

The fault current is

$$I_b = -j\sqrt{3} I_a^1 = -9.116 \ \text{pu}$$

10.13. Repeat Problem 10.11 for a bolted double line-to-ground fault on phases b and c.

The positive- and zero-sequence fault currents in phase a are

$$\begin{aligned} I_a^1 &= \frac{1}{j0.105 + j\left(\frac{(0.085)(0.06)}{0.085+0.06}\right)} = -j7.13407 \ \text{pu} \\ I_a^0 &= -\frac{1 - (j0.105)(-j7.13407)}{j0.06} = j4.182 \ \text{pu} \end{aligned}$$

The fault current is

$$I_f = 3I_a^0 = 12.546\angle 90^\circ$$

10.14. The zero-, positive-, and negative-sequence bus impedance matrices for a three-bus power system are

$$\mathbf{Z}_{bus}^0 = j \begin{bmatrix} 0.20 & 0.05 & 0.12 \\ 0.05 & 0.10 & 0.08 \\ 0.12 & 0.08 & 0.30 \end{bmatrix} \quad \text{pu}$$

$$\mathbf{Z}_{bus}^1 = \mathbf{Z}_{bus}^2 = j \begin{bmatrix} 0.16 & 0.10 & 0.15 \\ 0.10 & 0.20 & 0.12 \\ 0.15 & 0.12 & 0.25 \end{bmatrix} \quad \text{pu}$$

Determine the per unit fault current and the bus voltages during fault for
(a) A bolted three-phase fault at bus 2.
(b) A bolted single line-to-ground fault at bus 2.
(c) A bolted line-to-line fault at bus 2.
(d) A bolted double line-to-ground fault at bus 2.

(a) The symmetrical components of fault current for a bolted balanced three-phase fault at bus 2 is given by

$$I_2^{012}(F) = \begin{bmatrix} 0 \\ \frac{1}{Z_{22}^1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{j0.20} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5\angle{-90^\circ} \\ 5\angle 150^\circ \\ 5\angle 30^\circ \end{bmatrix}$$

For balanced fault we only have the positive-sequence component of voltage. Thus, from (10.98), bus voltages during fault for phase a are

$$\begin{aligned} V_1(F) &= 1 - Z_{12}^1 I_2(F) = 1 - j0.10(-j5) = 0.5 \\ V_2(F) &= 1 - Z_{22}^1 I_2(F) = 1 - j0.20(-j5) = 0.0 \\ V_3(F) &= 1 - Z_{32}^1 I_2(F) = 1 - j0.12(-j5) = 0.4 \end{aligned}$$

(b) From (10.90), the symmetrical components of fault current for a single line-to-ground fault at bus 2 is given by

$$\begin{aligned} I_2^0(F) = I_2^1(F) = I_2^2(F) &= \frac{1.0}{Z_{22}^1 + Z_{22}^2 + Z_{22}^0} \\ &= \frac{1.0}{j0.20 + j0.20 + j0.10} = -j2 \end{aligned}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j2 \\ -j2 \\ -j2 \end{bmatrix} = \begin{bmatrix} 6\angle{-90^\circ} \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(-j2) \\ 1 - j0.10(-j2) \\ 0 - j0.10(-j2) \end{bmatrix} = \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(-j2) \\ 1 - j0.20(-j2) \\ 0 - j0.20(-j2) \end{bmatrix} = \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(-j2) \\ 1 - j0.12(-j2) \\ 0 - j0.12(-j2) \end{bmatrix} = \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix} = \begin{bmatrix} 0.50\angle 0^\circ \\ 0.9539\angle -114.79^\circ \\ 0.9539\angle +114.79^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix} = \begin{bmatrix} 0.0\angle 0^\circ \\ 0.9165\angle -109.11^\circ \\ 0.9165\angle +109.11^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix} = \begin{bmatrix} 0.36\angle 0^\circ \\ 0.9625\angle -115.87^\circ \\ 0.9625\angle +115.87^\circ \end{bmatrix}$$

(c) From (10.92) and (10.93), the symmetrical components of fault current for line-to-line fault at bus 2 are

$$I_2^0 = 0$$
$$I_2^1 = -I_2^2 = \frac{V_2(0)}{Z_{22}^1 + Z_{22}^2} = \frac{1}{j0.20 + j0.20} = -j2.5$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.5 \\ j2.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.33 \\ 4.33 \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.10(-j2.5) \\ 0 - j0.10(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.20(-j2.50) \\ 0 - j0.20(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.12(-j2.5) \\ 0 - j0.12(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6614\angle -139.11^\circ \\ 0.614\angle +130.11^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.50\angle 180^\circ \\ 0.50\angle +180^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6083\angle -145.285^\circ \\ 0.6083\angle +145.285^\circ \end{bmatrix}$$

(d) From (10.94)–(10.96), the symmetrical components of fault current for a double line-to-ground fault at bus 2 is given by

$$I_2^1 = \frac{V_2(0)}{Z_{22}^1 + \frac{Z_{22}^2(Z_{22}^0)}{Z_{22}^2 + Z_{22}^0}} = -\frac{1}{j0.20 + \frac{j0.20(j0.10)}{j0.20 + j0.10}} = -j3.75$$

$$I_2^2 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^2} - \frac{1 - j0.20(-j3.75)}{j0.20} = j1.25$$

$$I_2^0 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^0} = -\frac{1 - j0.20(-j3.75)}{j0.20} = j2.5$$

The phase currents at the faulted bus are

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j2.5 \\ -j3.75 \\ j1.25 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.7282\angle 139.11^\circ \\ 5.7282\angle 40.89^\circ \end{bmatrix}$$

and the total fault current is

$$I_2^b + I_2^c = 5.7282\angle 139.11^\circ + 5.7282\angle 40.89^\circ = 7.5\angle 90^\circ$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(j2.5) \\ 1 - j0.10(-j3.75) \\ 0 - j0.10(j1.25) \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(j2.5) \\ 1 - j0.20(-j3.75) \\ 0 - j0.20(j1.25) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(j2.5) \\ 1 - j0.12(-j3.75) \\ 0 - j0.12(j1.25) \end{bmatrix} = \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 0.875\angle 0^\circ \\ 0.50\angle -120^\circ \\ 0.50\angle +120^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.75\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.90\angle 0^\circ \\ 0.3775\angle -113.413^\circ \\ 0.3775\angle +113.413^\circ \end{bmatrix}$$

10.15. The reactance data for the power system shown in Figure 85 in per unit on a common base is as follows:



FIGURE 85
The impedance diagram for Problem 10.15.

Obtain the Thévenin sequence impedances for the fault at bus 1 and compute the fault current in per unit for the following faults:

(a) A bolted three-phase fault at bus 1.
(b) A bolted single line-to-ground fault at bus 1.
(c) A bolted line-to-line fault at bus 1.
(d) A bolted double line-to-ground fault at bus 1.

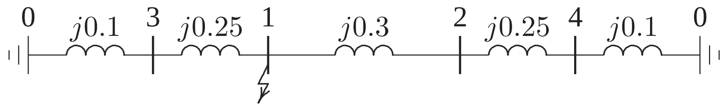


FIGURE 86
Positive-sequence impedance network for Problem 10.15.

The positive-sequence impedance network is shown in Figure 86, and impedance to the point of fault is

$$Z^1 = j\frac{(0.35)(0.65)}{0.35+0.65} = j0.2275 \text{ pu}$$

Since negative-sequence reactances are the same as positive-sequence reactances, $X^2 = X^1$ = 0.2275. The zero-sequence impedance network is shown in Figure 87, and impedance to the point of fault is

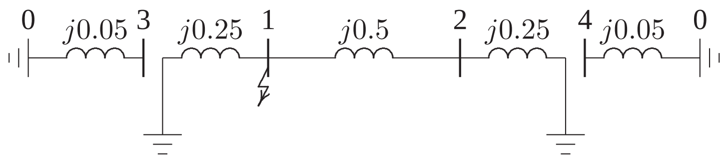


FIGURE 87
zero-sequence impedance network for Problem 10.15.

$$Z^0 = j\frac{(0.25)(0.75)}{0.25+0.75} = j0.1875 \text{ pu}$$

(a) For a bolted three-phase fault at bus 1, the fault current is

$$I_f = \frac{1}{j0.2275} = 4.3956\angle{-90^\circ} \text{ pu}$$

(b) For a bolted single-line to ground fault at bus 1, the fault current is

$$I_f = 3I_a^0 = \frac{3}{j(0.2275+0.2275+0.1875)} = 4.669\angle{-90^\circ} \text{ pu}$$

(c) For a bolted line-to-line fault at bus 1, the fault current in phase b is

$$I_a^1 = \frac{1}{j(0.2275+0.2275)} = -j2.1978 \text{ pu}$$

$$I_b(F) = -j\sqrt{3}I_a^1 = -3.8067 \text{ pu}$$

(d) For a bolted double line-to-line fault at bus 1, we have

$$I_a^1 = \frac{1}{j0.2275 + +j\frac{(0.2275)(0.1875)}{0.2275+0.1875}} = -j3.02767 \text{ pu}$$

$$I_a^0 = \frac{1-(0.2275)(-j3.02767)}{j0.1875} = j1.65975 \text{ pu}$$

$$I(F) = 3I_a^0 = 4.979\angle 90^\circ \text{ pu}$$

10.16. For Problem 10.15, obtain the bus impedance matrices for the sequence networks. A bolted single line-to-ground fault occurs at bus 1. Find the fault current, the three-phase bus voltages during fault, and the line currents in each phase. Check your results using the **zbuild** and **lgfault** programs.

First, we obtain the positive-sequence bus impedance matrix. Add branch 1, $z_{30} = j0.1$ between node $q = 3$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{33} = z_{30} = j0.1$$

Next, add branch 2, $z_{40} = j0.1$ between node $q = 4$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{33} & 0 \\ 0 & Z_{44} \end{bmatrix} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

Add branch 3, $z_{24} = j0.25$ between the new node $q = 2$ and the existing node $p = 4$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} j0.35 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix}$$

Add branch 4, $z_{13} = j0.25$ between the new node $q = 1$ and the existing node $p = 3$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.35 & 0 & j0.1 & 0 \\ 0 & j0.35 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & 0 \\ 0 & j0.1 & 0 & j0.1 \end{bmatrix}$$

Add link 5, $z_{12} = j0.3$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(5)} = \left[\begin{array}{cccc|c} j0.35 & 0 & j0.1 & 0 & -j0.35 \\ 0 & j0.35 & 0 & j0.1 & j0.35 \\ j0.1 & 0 & j0.1 & 0 & -j0.1 \\ 0 & j0.1 & 0 & j0.1 & j0.1 \\ \hline -j0.35 & j0.35 & -j0.1 & j0.1 & j1 \end{array}\right]$$

From (9.58)

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j1}\begin{bmatrix} -j0.35 \\ j0.35 \\ -j0.1 \\ j0.1 \end{bmatrix}\left[\, -j0.35 \;\; j0.35 \;\; -j0.1 \;\; j0.1 \,\right]$$

$$= \begin{bmatrix} j0.1225 & -j0.1225 & j0.0350 & -j0.0350 \\ -j0.1225 & j0.1225 & -j0.0350 & j0.0350 \\ j0.0350 & -j0.0350 & j0.0100 & -j0.0100 \\ -j0.0350 & j0.0350 & -j0.0100 & j0.0100 \end{bmatrix}$$

From (9.59), the new positive-sequence bus impedance matrix is

$$\mathbf{Z}_{bus}^{1} = \begin{bmatrix} j0.35 & 0 & j0.1 & 0 \\ 0 & j0.35 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & 0 \\ 0 & j0.1 & 0 & j0.1 \end{bmatrix} - \begin{bmatrix} j0.1225 & -j0.1225 & j0.0350 & -j0.0350 \\ -j0.1225 & j0.1225 & -j0.0350 & j0.0350 \\ j0.0350 & -j0.0350 & j0.0100 & -j0.0100 \\ -j0.0350 & j0.0350 & -j0.0100 & j0.0100 \end{bmatrix}$$

$$= \begin{bmatrix} j0.2275 & j0.1225 & j0.0650 & j0.0350 \\ j0.1225 & j0.2275 & j0.0350 & j0.0650 \\ j0.0650 & j0.0350 & j0.0900 & j0.0100 \\ j0.0350 & j0.0650 & j0.0100 & j0.0900 \end{bmatrix}$$

Next, we obtain the zero-sequence bus impedance matrix. Add branch 1, $z_{10} = j0.25$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.25$$

Next, add branch 2, $z_{20} = j0.25$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix}$$

Add link 3, $z_{12} = j0.5$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(3)} = \left[\begin{array}{cc|c} j0.25 & 0 & -j0.25 \\ 0 & j0.25 & j0.25 \\ \hline -j0.25 & j0.25 & j1 \end{array}\right]$$

From (9.58)

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j1}\begin{bmatrix} -j0.25 \\ j0.25 \end{bmatrix}\left[\, -j0.25 \;\; j0.25 \,\right]$$

$$= \begin{bmatrix} j0.0625 & -j0.0625 \\ -j0.0625 & j0.0625 \end{bmatrix}$$

From (9.59), the new positive-sequence bus impedance matrix is

$$\begin{aligned}\mathbf{Z}_{bus}^{0} &= \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix} - \begin{bmatrix} j0.0625 & -j0.0625 \\ -j0.0625 & j0.0625 \end{bmatrix} \\ &= \begin{bmatrix} j0.1875 & j0.0625 \\ j0.0625 & j0.1875 \end{bmatrix}\end{aligned}$$

For a bolted single line-to-ground fault at bus 1, from (10.90), the symmetrical components of fault current is given by

$$\begin{aligned}I_1^0(F) = I_1^1(F) = I_1^2(F) &= \frac{1.0}{Z_{11}^1 + Z_{11}^2 + Z_{11}^0} \\ &= \frac{1.0}{j0.2275 + j0.2275 + j0.1875} = -j1.5564\end{aligned}$$

The fault current is

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j1.5564 \\ -j1.5564 \\ -j1.5564 \end{bmatrix} = \begin{bmatrix} 4.6693\angle{-90^\circ} \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{11}^0 I_1^0 \\ V_1^1(0) - Z_{11}^1 I_1^1 \\ 0 - Z_{11}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.1875(-j1.5564) \\ 1 - j0.2275(-j1.5564) \\ 0 - j0.2275(-j1.5564) \end{bmatrix} = \begin{bmatrix} -0.2918 \\ 0.6459 \\ -0.3541 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{21}^0 I_1^0 \\ V_2^1(0) - Z_{21}^1 I_1^1 \\ 0 - Z_{21}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.0625(-j1.5564) \\ 1 - j0.1225(-j1.5564) \\ 0 - j0.1225(-j1.5564) \end{bmatrix} = \begin{bmatrix} -0.0973 \\ 0.8093 \\ -0.1907 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{31}^0 I_1^0 \\ V_3^1(0) - Z_{31}^1 I_1^1 \\ 0 - Z_{31}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0(-j1.5564) \\ 1 - j0.0650(-j1.5564) \\ 0 - j0.0650(-j1.5564) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8988 \\ -0.1012 \end{bmatrix}$$

$$V_4^{012}(F) = \begin{bmatrix} 0 - Z_{41}^0 I_1^0 \\ V_4^1(0) - Z_{41}^1 I_1^1 \\ 0 - Z_{41}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0(-j1.5564) \\ 1 - j0.0350(-j1.5564) \\ 0 - j0.0350(-j1.5564) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9455 \\ -0.0545 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.2918 \\ 0.6459 \\ -0.3541 \end{bmatrix} = \begin{bmatrix} 0\angle{-180^\circ} \\ 0.9704\angle{-116.815^\circ} \\ 0.9704\angle{+116.815^\circ} \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0973 \\ 0.8093 \\ -0.1907 \end{bmatrix} = \begin{bmatrix} 0.5214\angle 0^\circ \\ 0.9567\angle -115.151^\circ \\ 0.9567\angle +115.151^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8988 \\ -0.1012 \end{bmatrix} = \begin{bmatrix} 0.7977\angle 0^\circ \\ 0.9535\angle -114.727^\circ \\ 0.9535\angle +114.727^\circ \end{bmatrix}$$

$$V_4^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.9455 \\ -0.0545 \end{bmatrix} = \begin{bmatrix} 0.8911\angle 0^\circ \\ 0.9739\angle -117.223^\circ \\ 0.9739\angle +117.7223^\circ \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{21}^{012} = \begin{bmatrix} \frac{V_2^0(F)-V_1^0(F)}{z_{12}^0} \\ \frac{V_2^1(F)-V_1^1(F)}{z_{12}^1} \\ \frac{V_2^2(F)-V_1^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} \frac{-0.0973-(-0.2918)}{j0.5} \\ \frac{0.8093-0.6459)}{j0.3} \\ \frac{-0.1907-(-0.3541)}{j0.3} \end{bmatrix} = \begin{bmatrix} 0.3891\angle -90^\circ \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix}$$

$$I_{31}^{012} = \begin{bmatrix} \frac{V_3^0(F)-V_1^0(F)}{z_{13}^0} \\ \frac{V_3^1(F)-V_1^1(F)}{z_{13}^1} \\ \frac{V_3^2(F)-V_1^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} \frac{0-(-0.2918)}{\infty} \\ \frac{0.8988-0.6459)}{j0.25} \\ \frac{-0.1012-(-0.3541)}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0117\angle -90^\circ \\ 1.0117\angle -90^\circ \end{bmatrix}$$

$$I_{42}^{012} = \begin{bmatrix} \frac{V_4^0(F)-V_2^0(F)}{z_{24}^0} \\ \frac{V_4^1(F)-V_2^1(F)}{z_{24}^1} \\ \frac{V_4^2(F)-V_2^2(F)}{z_{24}^2} \end{bmatrix} = \begin{bmatrix} \frac{0-(-0.0973)}{\infty} \\ \frac{0.9455-0.8093)}{j0.25} \\ \frac{-0.0545-(-0.1907)}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix}$$

The line fault currents are

$$I_{21}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.3891\angle -90^\circ \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 1.4784\angle -90^\circ \\ 0.1556\angle 90^\circ \\ 0.1556\angle 90^\circ \end{bmatrix}$$

$$I_{31}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.0117\angle 90^\circ \\ 1.0117\angle 90^\circ \end{bmatrix} = \begin{bmatrix} 2.0233\angle -90^\circ \\ 1.0117\angle 90^\circ \\ 1.0117\angle 90^\circ \end{bmatrix}$$

$$I_{42}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5447\angle{-90^\circ} \\ 0.5447\angle{-90^\circ} \end{bmatrix} = \begin{bmatrix} 1.0895\angle{-90^\circ} \\ 0.5447\angle{90^\circ} \\ 0.5447\angle{90^\circ} \end{bmatrix}$$

To check the result we use the following commands

```
zdata1 = [0   3   0    0.10
          0   4   0    0.10
          1   3   0    0.25
          1   2   0    0.3
          2   4   0    0.25];
zdata0 = [0   3   0    0.05
      0   4   0    0.05
      0   1   0    0.25
      0   2   0    0.25
      1   3   9999 9999
      1   2   0    0.50
      2   4   9999 9999];
Zbus1 = zbuild(zdata1)
Zbus0 = zbuild(zdata0)
zdata2 = zdata1;
Zbus2 = Zbus1;
lgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
```

which result in

```
Zbus1 =
        0 + 0.2275i   0 + 0.1225i   0 + 0.0650i   0 + 0.0350i
        0 + 0.1225i   0 + 0.2275i   0 + 0.0350i   0 + 0.0650i
        0 + 0.0650i   0 + 0.0350i   0 + 0.0900i   0 + 0.0100i
        0 + 0.0350i   0 + 0.0650i   0 + 0.0100i   0 + 0.0900i
Zbus0 =
        0 + 0.1875i   0 + 0.0625i   0 + 0.0000i   0 + 0.0000i
        0 + 0.0625i   0 + 0.1875i   0 + 0.0000i   0 + 0.0000i
        0 + 0.0000i   0 + 0.0000i   0 + 0.0500i   0 + 0.0000i
        0 + 0.0000i   0 + 0.0000i   0 + 0.0000i   0 + 0.0500i

Line-to-ground fault analysis
Enter Faulted Bus No. -> 1

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Single line to-ground fault at bus No. 1
Total fault current =    4.6693 per unit
```

```
Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      0.0000      0.9704      0.9704
     2      0.5214      0.9567      0.9567
     3      0.7977      0.9535      0.9535
     4      0.8911      0.9739      0.9739

Line currents for fault at bus No.  1

     From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
      1         F      4.6693      0.0000      0.0000
      2         1      1.4786      0.1556      0.1556
      3         1      2.0234      1.0117      1.0117
      4         2      1.0895      0.5447      0.5447
```

10.17. Repeat Problem 10.16 for a bolted line-to-line fault. Check your results using the **zbuild** and **llfault** programs.

For a line-to-line fault at bus 1, from (10.92) and (10.93), the symmetrical components of fault current are

$$I_1^0 = 0$$
$$I_1^1 = -I_1^2 = \frac{V_1(0)}{Z_{11}^1 + Z_{11}^2} = \frac{1}{j0.2275 + j0.2275} = -j2.1978$$

The fault current is

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.1978 \\ j2.1978 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.8067 \\ 3.8067 \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 \\ V_1^1(0) - Z_{11}^1 I_1^1 \\ 0 - Z_{11}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.2275(-j2.1978) \\ 0 - j0.2275(j2.1978) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 \\ V_2^1(0) - Z_{21}^1 I_1^1 \\ 0 - Z_{21}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.1225(-j2.1978) \\ 0 - j0.1225(j2.1978) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7308 \\ 0.2692 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 \\ V_3^1(0) - Z_{31}^1 I_1^1 \\ 0 - Z_{31}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.0650(-j2.1978) \\ 0 - j0.0650(j2.1978) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8571 \\ 0.1429 \end{bmatrix}$$

$$V_4^{012}(F) = \begin{bmatrix} 0 \\ V_4^1(0) - Z_{41}^1 I_1^1 \\ 0 - Z_{41}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.0350(-j2.1978) \\ 0 - j0.0350(j2.1978) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9231 \\ 0.0769 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.50\angle 180^\circ \\ 0.50\angle +180^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.7308 \\ 0.2692 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6401\angle -141.361^\circ \\ 0.6401\angle +141.361^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8571 \\ 0.1429 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.7954\angle -128.948^\circ \\ 0.7954\angle +128.948^\circ \end{bmatrix}$$

$$V_4^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.9231 \\ 0.0769 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.8871\angle -124.306^\circ \\ 0.8871\angle +124.306^\circ \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{21}^{012} = \begin{bmatrix} 0 \\ \frac{V_2^1(F) - V_1^1(F)}{z_{12}^1} \\ \frac{V_2^2(F) - V_1^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{0.7308 - 0.50}{j0.3} \\ \frac{0.2692 - 0.50}{j0.3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7692\angle -90^\circ \\ 0.7692\angle +90^\circ \end{bmatrix}$$

$$I_{31}^{012} = \begin{bmatrix} 0 \\ \frac{V_3^1(F) - V_1^1(F)}{z_{13}^1} \\ \frac{V_3^2(F) - V_1^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{0.8571 - 0.50}{j0.25} \\ \frac{0.1429 - 0.50}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.4286\angle -90^\circ \\ 1.4286\angle +90^\circ \end{bmatrix}$$

$$I_{42}^{012} = \begin{bmatrix} 0 \\ \frac{V_4^1(F)-V_2^1(F)}{z_{42}^1} \\ \frac{V_4^2(F)-V_2^2(F)}{z_{42}^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{0.9231-0.7308}{j0.25} \\ \frac{0.0769-0.2692}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7692\angle-90^\circ \\ 0.7692\angle+90^\circ \end{bmatrix}$$

The line fault currents are

$$I_{21}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.7692\angle-90^\circ \\ 0.7692 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.3323 \\ 1.3323 \end{bmatrix}$$

$$I_{31}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.4286\angle-90^\circ \\ 1.4286\angle+90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ -2.4744 \\ 2.4744 \end{bmatrix}$$

$$I_{42}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.7692\angle-90^\circ \\ 0.7692\angle+90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ -1.3323 \\ 1.3323 \end{bmatrix}$$

To check the result, add the following commands to the *MATLAB* statements in Problem 10.16

```
llfault(zdata1, Zbus1, zdata2, Zbus2)
```

which result in

```
Enter Faulted Bus No. -> 1
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Line-to-line fault at bus No. 1
Total fault current =    3.8067 per unit

Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      1.0000      0.5000      0.5000
     2      1.0000      0.6401      0.6401
     3      1.0000      0.7954      0.7954
     4      1.0000      0.8871      0.8871

Line currents for fault at bus No.  1
```

```
From      To      -----Line Current Magnitude----
Bus       Bus     Phase a     Phase b     Phase c
 1         F      0.0000      3.8067      3.8067
 2         1      0.0000      1.3323      1.3323
 3         1      0.0000      2.4744      2.4744
 4         2      0.0000      1.3323      1.3323
```

10.18. Repeat Problem 10.16 for a bolted double line-to-ground fault. Check your results using the **zbuild** and **dlgfault** programs.

For a double line-to-ground fault at bus 1, from (10.94)–(10.96), the symmetrical components of fault current is given by

$$I_1^1 = \frac{V_1(0)}{Z_{11}^1 + \frac{Z_{11}^2(Z_{11}^0)}{Z_{11}^2+Z_{11}^0}} = -\frac{1}{j0.2275 + \frac{j0.2275(j0.1875)}{j0.2275+j0.1875}} = -j3.0277$$

$$I_1^2 = -\frac{V_1(0) - Z_{11}^1 I_1^1}{Z_{11}^2} - \frac{1 - j0.2275(-j3.0277)}{j0.2275} = j1.3679$$

$$I_1^0 = -\frac{V_1(0) - Z_{11}^1 I_1^1}{Z_{11}^0} = -\frac{1 - j0.2275(-j3.0277)}{j0.1875} = j1.6598$$

The phase currents at the faulted bus are

$$I_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j1.6598 \\ -j3.0277 \\ j1.3679 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.5485\angle 146.815° \\ 4.5485\angle 33.185° \end{bmatrix}$$

and the total fault current is

$$I_3^b + I_3^c = 4.5485\angle 146.815° + 4.5485\angle 33.185° = 4.9793\angle 90°$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{11}^0 I_1^0 \\ V_1^1(0) - Z_{11}^1 I_1^1 \\ 0 - Z_{11}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.1875(j1.6598) \\ 1 - j0.2275(-j3.0277) \\ 0 - j0.2275(j1.3679) \end{bmatrix} = \begin{bmatrix} 0.3112 \\ 0.3112 \\ 0.3112 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{21}^0 I_1^0 \\ V_2^1(0) - Z_{21}^1 I_1^1 \\ 0 - Z_{21}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.0625(j1.6598) \\ 1 - j0.1225(-j3.0277 \\ 0 - j0.1225(j1.3679) \end{bmatrix} = \begin{bmatrix} 0.1037 \\ 0.6291 \\ 0.1676 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{31}^0 I_1^0 \\ V_3^1(0) - Z_{31}^1 I_1^1 \\ 0 - Z_{31}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - 0(j1.6598) \\ 1 - j0.0650(-j3.0277) \\ 0 - j0.0650(j1.3679) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8032 \\ 0.0889 \end{bmatrix}$$

$$V_4^{012}(F) = \begin{bmatrix} 0 - Z_{41}^0 I_1^0 \\ V_4^1(0) - Z_{41}^1 I_1^1 \\ 0 - Z_{41}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - 0(j1.6598) \\ 1 - j0.0350(-j3.0277) \\ 0 - j0.0350(j1.3679) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8940 \\ 0.0479 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.3112 \\ 0.3112 \\ 0.3112 \end{bmatrix} = \begin{bmatrix} 0.9336\angle 0° \\ 0 \\ 0 \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.1037 \\ 0.6291 \\ 0.1676 \end{bmatrix} = \begin{bmatrix} 0.9004\angle 0° \\ 0.4965\angle -126.39° \\ 0.4965\angle +126.39° \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8032 \\ 0.0889 \end{bmatrix} = \begin{bmatrix} 0.8921\angle 0° \\ 0.7626\angle -125.795° \\ 0.7626\angle +125.795° \end{bmatrix}$$

$$V_4^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8940 \\ 0.0479 \end{bmatrix} = \begin{bmatrix} 0.9419\angle 0° \\ 0.8711\angle -122.728° \\ 0.8711\angle +122.728° \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{21}^{012} = \begin{bmatrix} \frac{V_2^0(F) - V_1^0(F)}{z_{12}^0} \\ \frac{V_2^1(F) - V_1^1(F)}{z_{12}^1} \\ \frac{V_2^2(F) - V_1^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} \frac{0.1037 - 0.3112}{j0.5} \\ \frac{0.6291 - 0.3112)}{j0.3} \\ \frac{0.1676 - 0.3112}{j0.3} \end{bmatrix} = \begin{bmatrix} 0.4149\angle 90° \\ 1.0597\angle -90° \\ 0.4788\angle 90° \end{bmatrix}$$

$$I_{31}^{012} = \begin{bmatrix} \frac{V_3^0(F) - V_1^0(F)}{z_{13}^0} \\ \frac{V_3^1(F) - V_1^1(F)}{z_{13}^1} \\ \frac{V_3^2(F) - V_1^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} \frac{0 - 0.3112}{\infty} \\ \frac{0.8032 - 0.3112}{j0.25} \\ \frac{0.0889 - 0.3112}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.968\angle -90° \\ 0.8892\angle +90° \end{bmatrix}$$

$$I_{42}^{012} = \begin{bmatrix} \frac{V_4^0(F) - V_2^0(F)}{z_{42}^0} \\ \frac{V_4^1(F) - V_2^1(F)}{z_{42}^1} \\ \frac{V_4^2(F) - V_2^2(F)}{z_{42}^2} \end{bmatrix} = \begin{bmatrix} \frac{0 - 0.1037}{\infty} \\ \frac{0.894 - 0.6291}{j0.25} \\ \frac{0.0479 - 0.1676}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0597\angle -90° \\ 0.4788\angle +90° \end{bmatrix}$$

The line fault currents are

$$I_{21}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.4149\angle{+90^\circ} \\ 1.0597\angle{-90^\circ} \\ 0.4788\angle 90^\circ \end{bmatrix} = \begin{bmatrix} 0.1660\angle{-90^\circ} \\ 1.5076\angle 152.102 \\ 1.5076\angle 27.898^\circ \end{bmatrix}$$

$$I_{31}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.9680\angle{-90^\circ} \\ 0.8892\angle{+90^\circ} \end{bmatrix} = \begin{bmatrix} 1.0788\angle{-90^\circ} \\ 2.5325\angle 167.702^\circ \\ 2.5325\angle 12.298^\circ \end{bmatrix}$$

$$I_{42}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.05997\angle{-90^\circ} \\ 0.4788\angle{+90^\circ} \end{bmatrix} = \begin{bmatrix} 0.5809\angle{-90^\circ} \\ 1.3636\angle 167.702^\circ \\ 1.3636\angle 12.298^\circ \end{bmatrix}$$

To check the result, we add the following command to the *MATLAB* statements in Problem 10.16

```
dlgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
```

which result in

```
Double line-to-ground fault analysis
Enter Faulted Bus No. -> 1
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Double line-to-ground fault at bus No. 1
Total fault current =    4.9793 per unit

Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      0.9336      0.0000      0.0000
     2      0.9004      0.4965      0.4965
     3      0.8921      0.7626      0.7626
     4      0.9419      0.8711      0.8711

Line currents for fault at bus No.  1

     From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
      1         F      0.0000      4.5486      4.5485
      2         1      0.1660      1.5076      1.5076
      3         1      1.0788      2.5325      2.5325
      4         2      0.5809      1.3636      1.3636
```

10.19. The positive-sequence reactances for the power system shown in Figure 88 are in per unit on a common MVA base. Resistances are neglected and the negative-sequence impedances are assumed to be the same as the positive-sequence impedances. A bolted line-to-line fault occurs between phases b and c at bus 2. Before the fault occurrence, all bus voltages are 1.0 per unit. Obtain the positive-sequence bus impedance matrix. Find the fault current, the three-phase bus voltages during fault, and the line currents in each phase. Check your results using the **zbuild** and **llfault** programs.

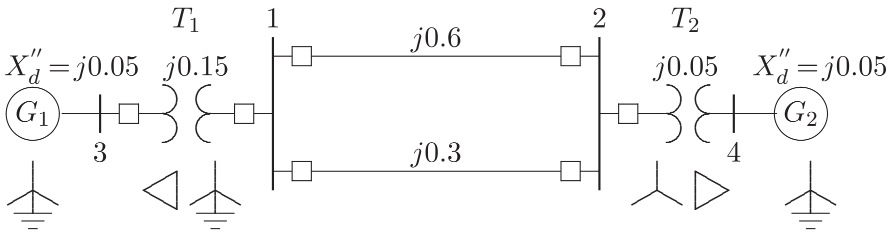


FIGURE 88
Circuit for Problem 10.19.

The following commands

```
zdata1 = [0   3   0    0.05
          0   4   0    0.05
          1   2   0    0.60
          1   2   0    0.30
          1   3   0    0.15
          2   4   0    0.05];
zdata2 = zdata1;
Zbus1 = zbuild(zdata1)
Zbus2 = Zbus1;
llfault(zdata1, Zbus1, zdata2, Zbus2)
```

result in

```
Zbus1 =
        0 + 0.1200i   0 + 0.0400i   0 + 0.0300i   0 + 0.0200i
        0 + 0.0400i   0 + 0.0800i   0 + 0.0100i   0 + 0.0400i
        0 + 0.0300i   0 + 0.0100i   0 + 0.0450i   0 + 0.0050i
        0 + 0.0200i   0 + 0.0400i   0 + 0.0050i   0 + 0.0450i

Line-to-line fault analysis
Enter Faulted Bus No. -> 2
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0
```

```
Line-to-line fault at bus No. 2
Total fault current =   10.8253 per unit

Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      1.0000      0.6614      0.6614
     2      1.0000      0.5000      0.5000
     3      1.0000      0.9079      0.9079
     4      1.0000      0.6614      0.6614

Line currents for fault at bus No.  2

     From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
      1         2      0.0000      0.7217      0.7217
      1         2      0.0000      1.4434      1.4434
      2         F      0.0000     10.8253     10.8253
      3         1      0.0000      2.1651      2.1651
      4         2      0.0000      8.6603      8.6603
Another fault location? Enter'y' or'n' within single quote->'n'
```

10.20. Use the **lgfault**, **llfault**, and **dlgfault** functions to compute the fault current, bus voltages, and line currents in the circuit given in Example 10.8 for the following unbalanced fault.
(a) A bolted single line-to-ground fault at bus 9.
(b) A bolted line-to-line fault at bus 9.
(c) A bolted double line-to-ground fault at bus 9.
All shunt capacitances and loads are neglected and the negative-sequence data is assumed to be the same as the positive-sequence data. All the prefault bus voltages are assumed to be unity.

Run chp10ex8 for a bolted fault at bus 9 **ch10p20**.

10.21. The six-bus power system network of an electric utility company is shown in Figure 89. The positive- and zero-sequence reactances of the lines and transformers in per unit on a 100-MVA base is tabulated below. The transformer connections are shown in Figure 89. The Δ-Y transformer between buses 3 and 5 is grounded through a reactor of reactance 0.10 per unit. The generator's positive- and zero-sequence reactances including the reactance of grounding neutrals on a 100-MVA base is tabulated below.

LINE AND TRANSFORMER DATA			
Bus No.	Bus No.	X^1, PU	X^0, PU
1	4	0.225	0.400
1	5	0.105	0.200
1	6	0.215	0.390
2	4	0.035	0.035
3	5	0.042	0.042
4	6	0.125	0.250
5	6	0.175	0.350

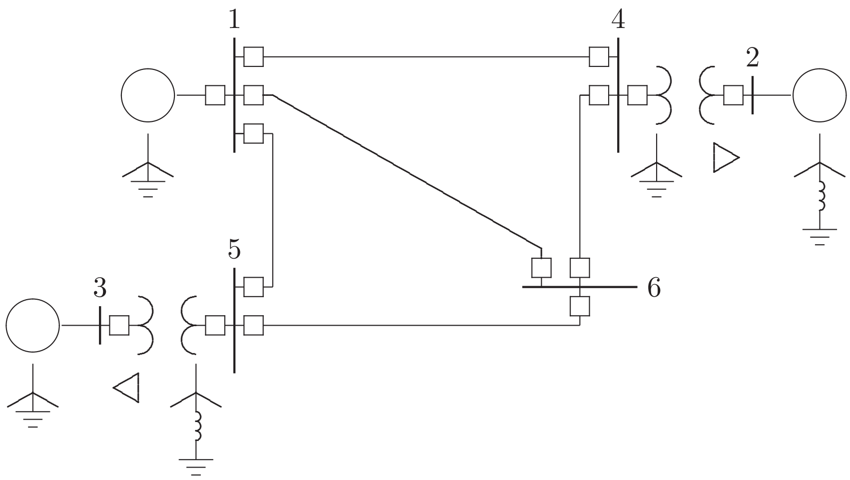


FIGURE 89
One-line diagram for Problem 10.21.

GENERATOR TRANSIENT IMPEDANCE, PU			
Gen. No.	X^1	X^0	X_n
1	0.20	0.06	0.00
2	0.15	0.04	0.05
3	0.25	0.08	0.00

Resistances, shunt reactances, and loads are neglected, and all negative-sequence reactances are assumed equal to the positive-sequence reactances. Use **zbuild** function to obtain the positive- and zero-sequence bus impedance matrices. Assume all the prefault bus voltages are equal to $1\angle 0°$, use **lgfault**, **llfault**, and **dlgfault** to compute the fault current, bus voltages, and line currents for the following unbalanced faults.

(a) A bolted single line-to-ground fault at bus 6.
(b) A bolted line-to-line fault at bus 6.
(c) A bolted double line-to-ground fault at bus 6.

The zero-sequence impedance network is as shown in Figure 90.

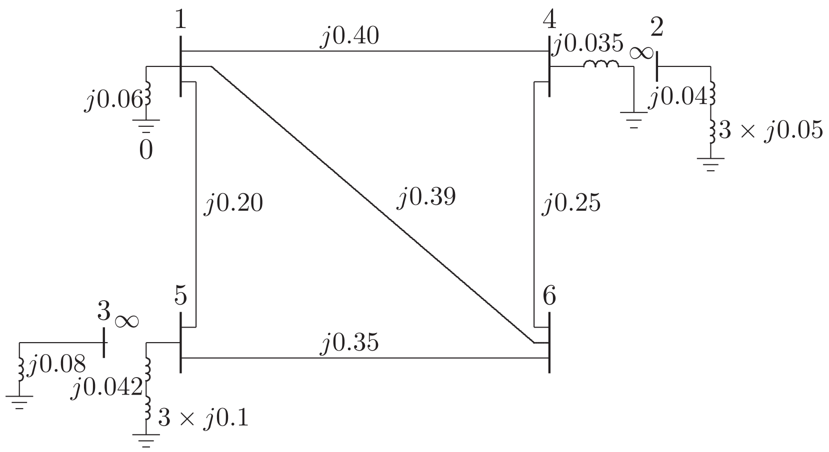


FIGURE 90
Zero-sequence impedance network for Problem 10.21.

The positive- and zero-sequence impedance data and the required commands are as follows.

```
zdata1 = [ 0     1     0.0     0.200
           0     2     0.0     0.150
           0     3     0.0     0.250
           1     4     0.0     0.225
           1     5     0.0     0.105
           1     6     0.0     0.215
           2     4     0.0     0.035
           3     5     0.0     0.042
           4     6     0.0     0.125
           5     6     0.0     0.175];

zdata0 = [ 0     1     0.0     0.060
           0     2     0.0     0.040+3*.05
           0     3     0.0     0.080
           0     4     0.0     0.035
           0     5     0.0     3*0.1
           1     4     0.0     0.400
           1     5     0.0     0.200
```

```
          1     6     0.0    0.390
          2     4     0.0     inf
          3     5     0.0     inf
          4     6     0.0    0.250
          5     6     0.0    0.350];
zdata2=zdata1;
Zbus0 = zbuild(zdata0)
Zbus1 = zbuild(zdata1)
Zbus2 = Zbus1;
lgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
llfault(zdata1, Zbus1,zdata2, Zbus2)
dlgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
```

The result is

```
Zbus0 =
       0.0449i   0.0000i   0.0000i   0.0058i   0.0258i   0.0225i
       0.0000i   0.1900i   0.0000i   0.0000i   0.0000i   0.0000i
       0.0000i   0.0000i   0.0800i   0.0000i   0.0000i   0.0000i
       0.0058i   0.0000i   0.0000i   0.0308i   0.0069i   0.0167i
       0.0258i   0.0000i   0.0000i   0.0069i   0.1121i   0.0439i
       0.0225i   0.0000i   0.0000i   0.0167i   0.0439i   0.1327i

Zbus1 =
       0.0940i   0.0440i   0.0591i   0.0543i   0.0691i   0.0689i
       0.0440i   0.0960i   0.0350i   0.0834i   0.0409i   0.0602i
       0.0591i   0.0350i   0.1177i   0.0432i   0.0955i   0.0635i
       0.0543i   0.0834i   0.0432i   0.1028i   0.0504i   0.0742i
       0.0691i   0.0409i   0.0955i   0.0504i   0.1115i   0.0742i
       0.0689i   0.0602i   0.0635i   0.0742i   0.0742i   0.1273i


Line-to-ground fault analysis
Enter Faulted Bus No. -> 6

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Single line to-ground fault at bus No. 6
Total fault current =    7.7449 per unit

Bus Voltages during the fault in per unit
```

```
Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      0.5859      0.9458      0.9458
     2      0.6892      0.9321      0.9321
     3      0.6721      0.9289      0.9289
     4      0.5735      0.9347      0.9347
     5      0.5036      0.9634      0.9634
     6      0.0000      1.0070      1.0070


Line currents for fault at bus No.  6

     From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
      1         4      0.0840      0.0979      0.0979
      1         6      2.1313      0.0284      0.0284
      2         4      2.0718      1.0359      1.0359
      3         5      1.3115      0.6557      0.6557
      4         6      3.3905      0.1010      0.1010
      5         1      0.5334      0.1482      0.1482
      5         6      2.2230      0.1294      0.1294
      6         F      7.7449      0.0000      0.0000
Another fault location? Enter'y' or'n' within single quote->'n'

Line-to-line fault at bus No. 6
Total fault current =    6.8017 per unit

Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      1.0000      0.6385      0.6385
     2      1.0000      0.6771      0.6771
     3      1.0000      0.6622      0.6622
     4      1.0000      0.6168      0.6168
     5      1.0000      0.6170      0.6170
     6      1.0000      0.5000      0.5000

Line currents for fault at bus No.  6

     From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
      1         4      0.0000      0.1597      0.1597
      1         5      0.0000      0.3383      0.3383
      1         6      0.0000      1.8468      1.8468
```

```
      2         4       0.0000       2.7292       2.7292
      3         5       0.0000       1.7277       1.7277
      4         6       0.0000       2.8890       2.8890
      5         6       0.0000       2.0660       2.0660
      6         F       0.0000       6.8017       6.8017

Another fault location? Enter'y' or'n' within single quote->'n'

Double line-to-ground fault analysis
Enter Faulted Bus No. -> 6

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Double line-to-ground fault at bus No. 6
Total fault current =    7.6388 per unit

Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      0.8818      0.5325      0.5325
     2      0.8467      0.6227      0.6227
     3      0.8383      0.6034      0.6034
     4      0.8536      0.5119      0.5119
     5      0.9230      0.4658      0.4658
     6      1.0137      0.0000      0.0000

Line currents for fault at bus No.  6

     From      To       -----Line Current Magnitude----
     Bus       Bus      Phase a      Phase b      Phase c
      1         4       0.0966       0.1599       0.1599
      1         5       0.1462       0.4769       0.4769
      1         6       0.0281       2.1319       2.1319
      2         4       1.0217       2.7766       2.7766
      3         5       0.6468       1.7577       1.7577
      4         6       0.0996       3.3632       3.3632
      5         6       0.1277       2.3096       2.3096
      6         F       0.0000       7.8007       7.8007

Another fault location? Enter 'y' or 'n' within single quote -> 'n'
```

CHAPTER 11 PROBLEMS

11.1. A four-pole, 60-Hz synchronous generator has a rating of 200 MVA, 0.8 power factor lagging. The moment of inertia of the rotor is 45,100 kg·m^2. Determine M and H.

$$n_s = \frac{120 f_0}{P} = 1800 \text{ rpm}$$
$$\omega_{sm} = \frac{2\pi n_s}{60} = 188.495 \text{ rad/sec}$$

From (11.10)

$$W_k = \frac{1}{2} J \omega_{sm}^2 = \frac{1}{2}(45100)(188.495)^2 = 801.2 \text{ MJ}$$

From (11.12)

$$M = \frac{2W_k}{\omega_{sm}} = \frac{2(801.2)}{188.495} = 8.5 \text{MJ sec}$$
$$H = \frac{W_k}{S_B} = \frac{801.2}{200} = 4 \text{ MJ/MVA}$$

11.2. A two-pole, 60-Hz synchronous generator has a rating of 250 MVA, 0.8 power factor lagging. The kinetic energy of the machine at synchronous speed is 1080 MJ. The machine is running steadily at synchronous speed and delivering 60 MW to a load at a power angle of 8 electrical degrees. The load is suddenly removed. Determine the acceleration of the rotor. If the acceleration computed for the generator is constant for a period of 12 cycles, determine the value of the power angle and the rpm at the end of this time.

$$H = \frac{W_k}{S_B} = \frac{1080}{250} = 4.32 \text{ MJ/MVA}$$

$$P_m = \frac{60}{250} = 0.24 \text{ pu}$$

$$12 \text{ cycle} = \frac{12}{60} = 0.2 \text{ sec}$$

From (11.22)

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = 0.24 - 0$$

or

$$\begin{aligned}\frac{d^2\delta}{dt^2} &= 600 \text{ degree/sec}^2\\ &= \frac{60}{360}(600) = 100 \text{ rpm/sec}\end{aligned}$$

Integrating the above expression, we have

$$\begin{aligned}\frac{d\delta}{dt} &= 600t\\ \delta &= \frac{1}{2}(600)t^2 + \delta_0\\ &= \frac{1}{2}(600)(0.2)^2 + 8 = 20^\circ\end{aligned}$$

$$n_s = \frac{120f_0}{P} = \frac{(120)(60)}{2} = 3600 \text{ rpm}$$

Speed at the end of 0.2 second is

$$n = 3600 + 0.2(100) = 3620 \text{ rpm}$$

11.3. Determine the kinetic energy stored by a 250-MVA, 60-Hz, two-pole synchronous generator with an inertia constant H of 5.4 MJ/MVA. Assume the machine is running steadily at synchronous speed with a shaft input of 331,100 hp The electrical power developed suddenly changes from its normal value to a value

of 200 MW. Determine the acceleration or deceleration of the rotor. If the acceleration computed for the generator is constant for a period of 9 cycles, determine the change in the power angle in that period and the rpm at the end of 9 cycles.

$$P_m = (331100)(746)(10^{-6}) = 247 \text{ MW}$$
$$= \frac{247}{250} = 0.988 \text{ pu}$$

$$P_e = \frac{200}{250} = 0.8 \text{ pu}$$

$$9 \text{ cycle} = \frac{9}{60} = 0.15 \text{ sec}$$

From (11.22), we have

$$\frac{5.4}{(180(60)} \frac{d^2\delta}{dt^2} = 0.988 - 0.8$$

or

$$\frac{d^2\delta}{dt^2} = 376 \text{ degree/sec}^2$$
$$= \frac{60}{360}(736) = 62.6667 \text{ rpm/sec}$$

Integrating the above expression, we have

$$\frac{d\delta}{dt} = 376t$$
$$\delta = \frac{1}{2}(376)t^2 + \delta_0$$
$$= \frac{1}{2}(376)(0.15)^2 + \delta_0$$

or

$$\Delta\delta = \delta - \delta_0 == 4.23^\circ$$

$$n_s = \frac{120f_0}{P} = \frac{(120)(60)}{2} = 3600 \text{ rpm}$$

Speed at the end of 0.15 second is

$$n = 3600 + 0.15(62.6667) = 3609.4 \text{ rpm}$$

11.4. The swing equations of two interconnected synchronous machines are written as

$$\frac{H_1}{\pi f_0}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$
$$\frac{H_2}{\pi f_0}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Obtain a swing equation equivalent to that of a single machine in terms of δ, and show that

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where

$$H = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} \quad \text{and} \quad P_e = \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}$$

Subtracting the second swing equation from the first, we obtain

$$\frac{1}{\pi f_0}\left(\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2}\right) = \left(\frac{P_{m1}}{H_1} - \frac{P_{m2}}{H_2}\right) - \left(\frac{P_{e1}}{H_1} - \frac{P_{e2}}{H_2}\right)$$

or

$$\frac{1}{\pi f_0}\left(\frac{d^2(\delta_1 - \delta_2)}{dt^2}\right) = \left(\frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 H_2}\right) - \left(\frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 H_2}\right)$$

Multiplying both side by $\frac{H_1 H_2}{H_1 + H_2}$, results in

$$\frac{1}{\pi f_0}\frac{H_1 H_2}{H_1 + H_2}\left(\frac{d^2(\delta_1 - \delta_2)}{dt^2}\right) = \left(\frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2}\right) - \left(\frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}\right)$$

or

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where H, δ, P_m, and P_e are defined above.

11.5. Two synchronous generators represented by a constant voltage behind transient reactance are connected by a pure reactance $X = 0.3$ per unit, as shown in Figure 91. The generator inertia constants are $H_1 = 4.0$ MJ/MVA and $H_2 = 6$ MJ/MVA, and the transient reactances are $X_1' = 0.16$ and $X_2' = 0.20$ per unit. The system is operating in the steady state with $E_1' = 1.2$, $P_{m1} = 1.5$ and $E_2' = 1.1$, $P_{m2} = 1.0$ per unit. Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Referring to Problem 11.4, reduce the two-machine system to an equivalent one-machine against an infinite bus. Find the inertia constant of the equivalent machine, the mechanical input power, and the amplitude of its power angle curve, and obtain the equivalent swing equation in terms of δ.

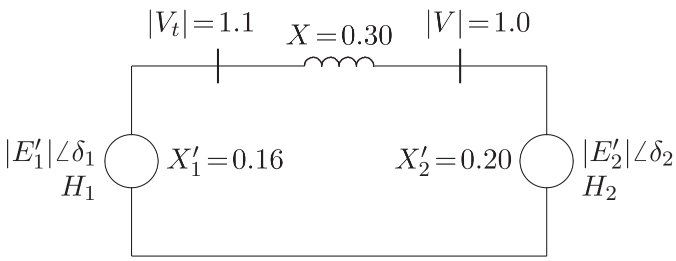


FIGURE 91
System of Problem 11.5.

Referring to Problem 11.4, the equivalent parameters are

$$H = \frac{(4)(6)}{4+6} = 2.4 \text{ MJ/MVA}$$

$$P_m = \frac{(6)(1.50) - (4)(1)}{4+6} = 0.5 \text{ pu}$$

$$P_{e1} = \frac{|E_1||E_2|}{X}\sin(\delta_1 - \delta_2) = \frac{(1.2)(1.1)}{0.66}\sin\delta = 2\sin\delta$$

Since $P_{e2} = -P_{e1}$, we have

$$P_e = \frac{(6)(2\sin\delta) + (4)(2\sin\delta)}{4+6} = 2\sin\delta$$

Therefore, the equivalent swing equation is

$$\frac{2.4}{(180)(60)}\frac{d^2\delta}{dt^2} = 0.5 - 2\sin\delta$$

or

$$\frac{d^2\delta}{dt^2} = 4500(0.5 - 2\sin\delta) \quad \text{where } \delta \text{ is in degrees}$$

11.6. A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown in Figure 92. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is 1.1. The infinite bus voltage $V = 1.0\angle 0°$ per unit. Determine the generator excitation voltage and obtain the swing equation as given by (pr11.36).

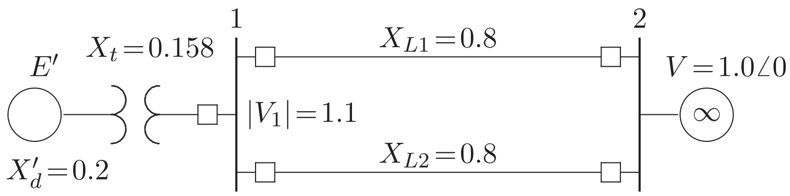


FIGURE 92
System of Problem 11.6.

$$P = \frac{|V_1||V_2|}{X_L} \sin \delta_1$$
$$0.77 = \frac{(1.1)(1.0)}{0.4} \sin \delta_1$$

or

$$\delta_1 = 16.26°$$

$$I = \frac{V_1 - V_2}{jX_L} = \frac{1.1\angle 16.26° - 1.0\angle 0°}{j0.4} = 0.77 - j0.14$$
$$= 0.7826\angle -10.305° \text{ pu}$$

The total reactance is $X = 0.2 + 0.158 + 0.4 = 0.758$, and the generator excitation voltage is

$$E' = 1.0 + j0.758(0.77 - j0.14) = 1.25\angle 27.819°$$

from (11.36) the swing equation with δ in radians is

$$\frac{5.66}{60\pi} \frac{d^2\delta}{dt^2} = 0.77 - \frac{(1.25)(1)}{0.758} \sin \delta$$
$$0.03 \frac{d^2\delta}{dt^2} = 0.77 - 1.65 \sin \delta$$

11.7. A three-phase fault occurs on the system of Problem 11.6 at the sending end of the transmission lines. The fault occurs through an impedance of 0.082 per unit. Assume the generator excitation voltage remains constant at $E' = 1.25$ per unit. Obtain the swing equation during the fault.

The impedance network with fault at bus 1, and with with $Z_f = j0.082$ is shown in Figure 93.

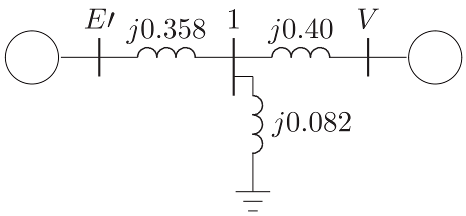


FIGURE 93
Impedance network with fault at bus 1.

Transforming the Y-connected circuit in Figure 93 into an equivalent Δ, the transfer reactance between E' and V is

$$X = \frac{(0.358)(0.082) + (0.358)(0.4) + (0.4)(0.082)}{0.082} = 2.5 \text{ pu}$$

$$P_{2\,max} = \frac{(1.25)(1)}{2.5} = 0.5$$

Therefore, the swing equation during fault with δ in radians is

$$0.03\frac{d^2\delta}{dt^2} \; = \; 0.77 - 0.5\sin\delta$$

11.8. The power-angle equation for a salient-pole generator is given by

$$P_e = P_{max}\sin\delta + P_K\sin 2\delta$$

Consider a small deviation in power angle from the initial operating point δ_0, i.e., $\delta = \delta + \delta_0$. Obtain an expression for the synchronizing power coefficient, similar to (11.39). Also, find the linearized swing equation in terms of $\Delta\delta$.

Substituting for δ in (11.36), we get

$$\frac{H}{\pi f_0}\frac{d^2(\delta_0 + \Delta\delta)}{dt^2} = P_m - P_{max}\sin(\delta_0 + \Delta\delta) - P_K\sin 2(\delta_0 + \Delta\delta)$$

or

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}(\sin\delta_0\cos\Delta\delta + \cos\delta_0\sin\Delta\delta) - P_K(\sin 2\delta_0\cos 2\Delta\delta + \cos 2\delta_0\sin 2\Delta\delta)$$

Since $\Delta\delta$ is small, $\cos 2\Delta\delta \cong 1$, $\cos\Delta\delta \cong 1$, $\sin 2\Delta\delta \cong 2\Delta\delta$ and $\sin\Delta\delta \cong \Delta\delta$, and we have

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}\sin\delta_0 - P_{max}\cos\delta_0\,(\Delta\delta) - P_K\sin 2\delta_0 - P_K\cos 2\delta_0\,(2\Delta\delta)$$

Since at the initial operating state

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} = P_m - P_{max}\sin\delta_0 - P_K\sin 2\delta_0$$

The above equation reduces to linearized equation in terms of incremental changes in power angle, i.e.,

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + (P_{max}\cos\delta_0 + 2P_K\cos 2\delta_0)\Delta\delta = 0$$

or

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_S\Delta\delta = 0$$

where P_S is the slope of the power-angle curve at δ_0, known as the *synchronizing power coefficient*, given by

$$P_s = \left.\frac{dP}{d\delta}\right|_{\delta_0} = P_{max}\cos\delta_0 + 2P_K\cos 2\delta_0$$

11.9. Consider the displacement x for a unit mass supported by a nonlinear spring as shown in Figure 94. The equation of motion is described by

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + K\sin x(t) = f(t)$$

where M is the mass, B is the frictional coefficient and K is the spring constant. The system is at the steady state $x(0) = 0$ for $f(0) = 0$. A small perturbation $f(t) = f(0) + \Delta f(t)$ results in the displacement $x(t) = x(0) + \Delta x(t)$.
(a) Obtain a linearized expression for the motion of the system in terms of the system parameters, $\Delta x(t)$ and $\Delta f(t)$.

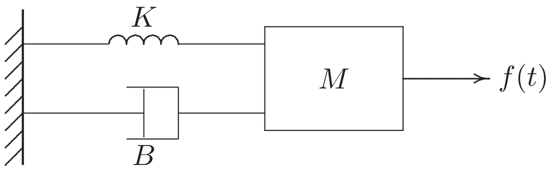


FIGURE 94
System of Problem 11.9.

(b) For $M = 1.6$, $B = 9.6$, and $K = 40$, find the damping ratio ζ and the damped frequency of oscillation ω_d.

(a) For a small deviation Δx, i.e., $x = x_0 + \Delta x$, the equation of motion becomes

$$(M\ddot{x}_0 + B\dot{x}_0 + K\sin x_0 \cos\Delta x) - f(t_0) + (M\ddot{\Delta x} + B\dot{\Delta x} + K\cos x_0 \sin\Delta x) = \Delta f(t)$$

Since Δx is small $\cos\Delta x \cong 1$ and $\sin\Delta x \cong \Delta x$, the first term becomes zero, and the above equation in terms of small changes in Δx becomes

$$M\ddot{\Delta x} + B\dot{\Delta x} + K\cos x_0\, \Delta x = \Delta f(t)$$

(b) For the given parameters, we have

$$1.6\ddot{\Delta x} + 9.6\dot{\Delta x} + 40\cos(0)\, \Delta x = \Delta f(t)$$

or

$$\ddot{\Delta x} + 6\dot{\Delta x} + 25\Delta x = \Delta f(t)$$

Therefore,

$$\begin{aligned} \omega_n^2 &= 25 \qquad \text{or} \qquad \omega_n = 5.0 \\ 2\zeta\omega_n &= \ \ 6 \qquad \text{or} \qquad \zeta = 0.6 \\ \omega_d &= \omega_n\sqrt{1-\zeta^2} = 5\sqrt{1-0.36} = 4.0 \end{aligned}$$

11.10. The machine in the power system of Problem 11.6 has a per unit damping coefficient of $D = 0.15$. The generator excitation voltage is $E' = 1.25$ per unit and the generator is delivering a real power of 0.77 per unit to the infinite bus at a voltage of $V = 1.0$ per unit. Write the linearized swing equation for this power system. Use (11.61) and (11.62) to find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta\delta = 15°$. Use

MATLAB to obtain the plots of rotor angle and frequency.

The initial operating angle is given by

$$0.77 = \frac{(1.25)(1)}{0.758}\sin\delta \qquad \text{or} \qquad \delta = 27.835^\circ = 0.4858 \text{ radian}$$

The synchronizing power coefficient given by (11.39) is

$$P_s = P_{max}\cos\delta_0 \; = \; \frac{(1.25)(1)}{0.758}\cos 27.835^\circ = 1.4583$$

The undamped angular frequency of oscillation and damping ratio are

$$\omega_n = \sqrt{\frac{\pi f_0}{H}P_s} = \sqrt{\frac{(\pi)(60)}{5.66}1.4583} = 6.969 \text{ rad/sec}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{HP_s}} = \frac{0.15}{2}\sqrt{\frac{(\pi)(60)}{(5.66)(1.4583)}} = 0.3584$$

The linearized force-free equation which determines the mode of oscillation given by (11.46) with δ in radian is

$$\frac{d^2\Delta\delta}{dt^2} + 5\frac{d\Delta\delta}{dt} + 48.565\Delta\delta = 0$$

From (11.50), the damped angular frequency of oscillation is

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.969\sqrt{1-(0.3584)^2} = 6.506 \text{ rad/sec}$$

corresponding to a damped oscillation frequency of

$$f_d = \frac{6.506}{2\pi} = 1.035 \text{ Hz}$$
$$\theta = \cos^{-1} 0.3584 = 69^\circ$$

From (11.61) and (11.62), the motion of rotor relative to the synchronously revolving field in electrical degrees and the frequency excursion in Hz are given by equations

$$\delta = 27.835^\circ + 16.0675e^{-2.4977t}\sin(6.506t + 69^\circ)$$
$$f = 60 - 0.311e^{-2.4977t}\sin 6.506t$$

Use the following commands to obtain the plots of rotor angle and frequency

```
t = 0:.01:3;
d=27.835+16.0675*exp(-2.4977*t).*sin(6.506*t+69*pi/180);
f = 60 - 0.311*exp(-2.4977*t).*sin(6.506*t);
figure(1), plot(t, d), grid
xlabel('t, sec.'), ylabel('Delta, degree')
figure(2) , plot(t,f), grid
xlabel('t, sec.'), ylabel('f, Hz')
```

11.11. Write the linearized swing equation of Problem 11.10 in state variable form. Use $[y, x]$ = **initial(**A, B, C, D, x_0, t**)** and **plot** commands to obtain the zero-input response for the initial conditions $\Delta\delta = 15°$, and $\Delta\omega_n = 0$.

From (11-52)–(11.55), we have

$$A = \begin{bmatrix} 0 & 1 \\ -48.5649 & -4.9955 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Use the following commands to obtain the plots of rotor angle and frequency

```
A = [0   1; -48.5649  -4.9955];
B = [0; 0];  C = [1 0; 0 1]; D = [0; 0];
Dd0 = 15*pi/180;                % Initial angle in radian
Dx0 = [Dd0; 0];                     % Initial conditions
t = 0:.01:3;
[y,x]= initial(A, B, C, D, Dx0, t);
Dd = x(:, 1); Dw = x(:, 2);   % State variables x1 and x2
d = 27.835 + Dd*180/pi;           % power angle in degree
f = 60 + Dw/(2*pi);                   % Frequency in Hz
figure(1), plot(t, d), grid
xlabel('t, sec'), ylabel('Delta, degree')
figure(2), plot(t, f), grid
xlabel('t, sec'), ylabel('f, Hz')
```

11.12. The generator of Problem 11.10 is operating in the steady state at $\delta_0 = 27.835°$ when the input power is increased by a small amount $\Delta P = 0.15$ per unit. The generator excitation and the infinite bus voltage are the same as before. Use (11.75) and (11.76) to find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta P = 0.15$ per unit. Use *MATLAB* to obtain the plots of rotor angle and frequency.

(a) Substituting for H, δ_0, ζ, and ω_n evaluated in Problem 11.11 in (11.75), and expressing the power angle in degree, we get

$$\delta = 27.835° + \frac{(180)(60)(0.15)}{(5.66)(48.565)}[1 - \frac{1}{\sqrt{1-(0.3548)^2}}e^{-2.4977t}\sin(6.506t + 69°)]$$

or

$$\delta = 27.835^\circ + 5.8935[1 - 1.0712e^{-2.4977t}\sin(6.506t + 69^\circ)]$$

Also, substituting the values in (11.76) and expressing the frequency in Hz, we get

$$f = 60 + \frac{(60)(0.15)}{2(5.66)(6.506)\sqrt{1-(0.3584)^2}}e^{-2.4977t}\sin 6.506t$$

or

$$f = 60 + 0.1222e^{-2.4977t}\sin 6.506t$$

Plot δ, and f over a range of 0 to 3 seconds.

11.13. Write the linearized swing equation of Problem 11.10 in state variable form. Use $[y, x]$ **= step(**$A, B, C, D, 1, t$**)** and **plot** commands to obtain the zero-state response when the input power is increased by a small amount $\Delta P = 0.15$ per unit.

From (11.70), the state equation for Problem 11.13 is

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -48.5649 & -4.9955 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta u$$

and

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

From (11.69), $\Delta u = (60\pi/5.66)(0.15) = 4.9955$. The following *MATLAB* commands are used to obtain the step response for Example 11.13.

```
Du = pi*60/5.66*0.15;  % Small step change in power input
A = [0  1; -48.5649  -4.9955];
B = [0;1]*Du;  C = [1 0; 0  1];
D = [0; 0];
t = 0:.01:3;
[y, x] = step(A, B, C, D, 1, t);
Dd = x(:, 1); Dw = x(:, 2);   % State variables x1 and x2
d = 27.835 + Dd*180/pi;           % Power angle in degree
f = 60 + Dw/(2*pi);                     % Frequency in Hz
figure(1), plot(t, d), grid
xlabel('t,  sec'), ylabel('Delta, degree')
figure(2), plot(t, f), grid
xlabel('t,  sec'), ylabel('f, Hz')
```

11.14. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit. Use **eacpower(**P_m, E, V, X**)** to find
(a) The maximum power input that can be added without loss of synchronism.
(b) Repeat (a) with zero initial power input. Assume the generator internal voltage remains constant at the value computed in (a).

In Problem 11.6, the transfer reactance and the generator internal voltage were found to be $X = 0.758$ pu, and $E' = 1.25$ pu.

We use the following commands

```
disp('(a) Initial real power P0 = 0.77')
P0 = 0.77; E = 1.25; V = 1.0; X = 0.758;
h=figure;
eacpower(P0, E, V, X)
h=figure;
disp('(b) Zero initial power ')
P0 = 0;
eacpower(P0, E, V, X)
```

which result in

```
(a) Initial real power P0 = 0.77
Initial power                     =  0.770 p.u.
Initial power angle               = 27.835 degrees
Sudden additional power           =  0.649 p.u.
Total power for critical stability =  1.419 p.u.
Maximum angle swing               =120.617 degrees
New operating angle               = 59.383 degrees
Current plot held
(b) Zero initial power
Initial power                     =  0.000 p.u.
Initial power angle               =  0.000 degrees
Sudden additional power           =  1.195 p.u.
Total power for critical stability =  1.195 p.u.
Maximum angle swing               =133.563 degrees
New operating angle               = 46.437 degrees
```

11.15. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit.
(a) A temporary three-phase fault occurs at the sending end of one of the transmission lines. When the fault is cleared, both lines are intact. Using equal area

criterion, determine the critical clearing angle and the critical fault clearing time. Use **eacfault(**P_m, E, V, X_1, X_2, X_3**)** to check the result and to display the power-angle plot.
(b) A three-phase fault occurs at the middle of one of the lines, the fault is cleared, and the faulted line is isolated. Determine the critical clearing angle. Use **eacfault (**P_m, E, V, X_1, X_2, X_3**)** to check the results and to display the power-angle plot.

(a) For a bolted fault at bus 1, the transfer reactance are

Reactance before the fault $X_1 = 0.758$
Reactance during the fault $X_2 = \infty$
Reactance after the fault $X_3 = 0.758$

(b) A bolted fault at the middle of one transmission line results in the circuit shown in Figure (11.5).

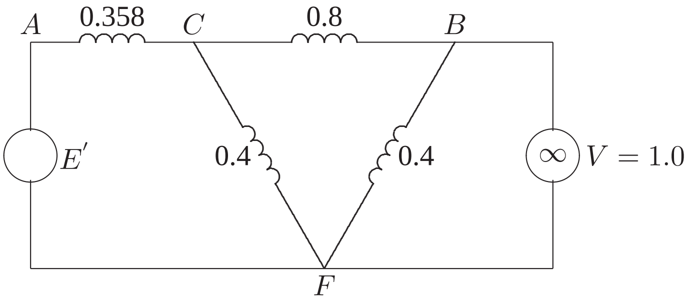


FIGURE 95
Equivalent circuit with three-phase fault at the middle of one line.

Converting the Y-circuit ABF to an equivalent δ, and eliminating junction C, the transfer reactance during the fault is

$$X_2 = \frac{(0.358)(0.8) + (0.358)(0.4) + (0.8)(0.4)}{0.4} = 1.874 \text{ pu}$$

From Figure (11.2), the reactance when the faulted line is isolated is $X_3 = 1.158$ pu.

We use the following commands

```
disp('(a) Fault at the sending end, ...
both lines intact when fault is cleared')
Pm = 0.77; E= 1.25; V = 1.0; X1 = 0.758; X2 = inf; X3 = 0.758;
H = 5.66;
P1max=E*V/X1; P2max=E*V/X2; P3max=E*V/X3;
d0=asin(Pm/P1max); dmax=pi-d0;
```

```
dc=acos((Pm*(dmax-d0)+P3max*cos(dmax)-P2max*cos(d0))/ ...
(P3max-P2max));
dcdeg=dc*180/pi;
disp(['Critical clearing angle =  ', num2str(dcdeg)])
tc = sqrt(2*H*(dc-d0)/(pi*60*Pm));
disp(['Critical clearing time = ', num2str(tc)])
% Check using eacfault
eacfault(Pm, E, V, X1, X2, X3)

disp('(b) Fault at the mid-point of one line, ....
faulted line is isolated')
X2 = 1.874; X3 = 1.158;
eacfault(Pm, E, V, X1, X2, X3)
```

which result in

```
(a) Fault at the sending end,
 both lines intact when fault is cleared
Critical clearing angle =  82.5927
Critical clearing time = 0.27302

For this case tc can be found from analytical formula.
To find tc enter Inertia Constant H, (or 0 to skip) H = 5.66

Initial power angle     =  27.835
Maximum angle swing     = 152.165
Critical clearing angle =  82.593

Critical clearing time  =   0.273 sec.
Current plot held

(b) Fault at the mid-point of one line,
faulted line is isolated

Initial power angle     =  27.835
Maximum angle swing     = 134.494
Critical clearing angle =  77.817
```

11.16. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit. A three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.
(a) The fault is cleared in 0.2 second. Obtain the numerical solution of the swing equation for 1.5 seconds. Select one of the functions **swingmeu**, **swingrk2**, or **swingrk4**.

(b) Repeat the simulation and obtain the swing plots when fault is cleared in 0.4 second, and for the critical clearing time.

The reactances between the generator internal emf and the infinite bus calculated in Problem 11.15 are: $X_1 = 0.758$, $X_2 = 1.874$, and $X_3 = 1.158$. The swing curves are obtained for fault clearing times of $t_c = 0.2$, $t_c = 0.4$, and $t_c = 0.37$, with **swingmeu** function. We use the following statements.

```
global  Pm f H E V X1 X2 X3
Pm = 0.770;  E = 1.25;  V = 1.0;
X1 = 0.758; X2 = 1.874; X3 = 1.158;
H = 5.66; f = 60; tf = 1.5; Dt = 0.01;
% (a)  Fault is cleared in 0.3 sec.
tc = 0.2;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
% (b) Fault is cleared in 0.4 sec.
tc = .4;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
tc = .37;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
```

Run the program to obtain the swing curves.

```
Critical clearing time =  0.37 seconds
Critical clearing angle =  77.82 degrees
```

11.17. Consider the power system network of Example 11.7 with the described operating condition. A three-phase fault occurs on line 1–5 near bus 5 and is cleared by the simultaneous opening of breakers at both ends of the line. Using the **trstab** program, perform a transient stability analysis. Determine the system stability for

(a) When the fault is cleared in 0.2 second
(b) When the fault is cleared in 0.4 second
(c) Repeat the simulation to determine the critical clearing time.

Run Example 11.7 for fault at bus 5 to obtain the swing plots for $t_c = 0.2$, and $t_c = 0.4$. Repeat until the critical clearing time $t_c = 0.34$ second is obtained.

11.18. The power system network of an electric company is shown in Figure 96.

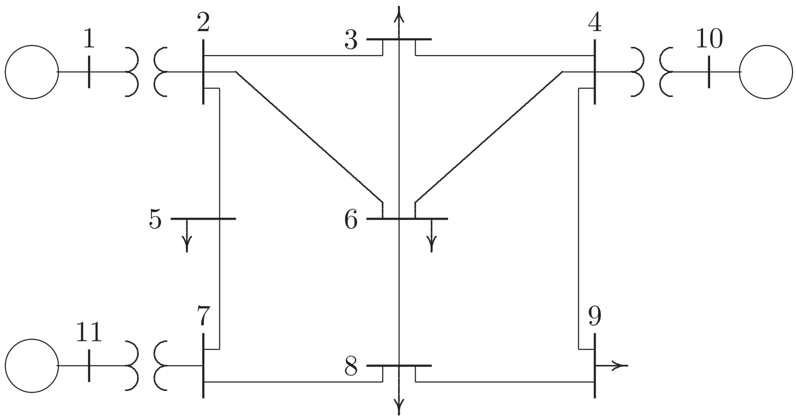


FIGURE 96
System of Problem 11.18.

The load data is as follows.

LOAD DATA					
Bus No.	Load MW	Load Mvar	Bus No.	Load MW	Load Mvar
1	0.0	0.0	7	0.0	0.0
2	0.0	0.0	8	110.0	90.0
3	150.0	120.0	9	80.0	50.0
4	0.0	0.0	10	0.0	0.0
5	120.0	60.0	11	0.0	0.0
6	140.0	90.0			

Voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.04\angle 0°$, is taken as the slack bus.

GENERATION SCHEDULE				
Bus No.	Voltage Mag.	Generation, MW	Mvar Limits Min.	Mvar Limits Max.
1	1.040			
10	1.035	200.0	0.0	180.0
11	1.030	160.0	0.0	120.0

The line data containing the per unit series impedance, and one-half of the shunt capacitive susceptance on a 100-MVA base is tabulated below.

LINE AND TRANSFORMER DATA				
Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}B$, PU
1	2	0.000	0.006	0.000
2	3	0.008	0.030	0.004
2	5	0.004	0.015	0.002
2	6	0.012	0.045	0.005
3	4	0.010	0.040	0.005
3	6	0.004	0.040	0.005
4	6	0.015	0.060	0.008
4	9	0.018	0.070	0.009
4	10	0.000	0.008	0.000
5	7	0.005	0.043	0.003
6	8	0.006	0.048	0.000
7	8	0.006	0.035	0.004
7	11	0.000	0.010	0.000
8	9	0.005	0.048	0.000

The generator's armature resistance and transient reactances in per unit, and the inertia constants expressed on a 100-MVA base are given below.

MACHINE DATA			
Gen.	R_a	X'_d	H
1	0	0.20	12
10	0	0.15	10
11	0	0.25	9

A three-phase fault occurs on line 4–9, near bus 4, and is cleared by the simultaneous opening of breakers at both ends of the line. Using the **trstab** program, perform a transient stability analysis. Determine the stability for
(a) When the fault is cleared in 0.4 second
(b) When the fault is cleared in 0.8 second
(c) Repeat the simulation to determine the critical clearing time.

The required data and commands are as follows:

```
basemva = 100;  accuracy = 0.0001;  maxiter = 10;
%       Bus Bus  |V|  Ang --Load---   --Gen---  Gen MVAR  Shunt
%       No. code p.u. Deg  MW    MVAR  MW  MVAR Min  MAx  React
busdata=[1   1   1.04  0  00.0   0.0   0.0  0.0   0    0    0
         2   0   1.0   0  00.0   0.0   0.0  0.0   0    0    0
         3   0   1.0   0 150.0 120.0   0.0  0.0   0    0    0
```

```
         4  0   1.0   0   0.0   0.0   0.0  0.0  0    0    0
         5  0   1.0   0 120.0  60.0   0.0  0.0  0    0    0
         6  0   1.0   0 140.0  90.0   0.0  0.0  0    0    0
         7  0   1.0   0   0.0   0.0   0.0  0.0  0    0    0
         8  0   1.0   0 110.0  90.0   0.0  0.0  0    0    0
         9  0   1.0   0  80.0  50.0   0.0  0.0  0    0    0
         10 2   1.035 0   0.0   0.0 200.0  0.0  0   180   0
         11 2   1.03  0   0.0   0.0 160.0  0.0  0   120   0];
%        Bus   Bus     R        X       1/2B     Line code or
%        No.   No.    p.u.     p.u.     p.u.  transformer tap
linedata=[1     2     00.00    0.006     0.000    1
          2     3     0.008    0.030     0.004    1
          2     5     0.004    0.015     0.002    1
          2     6     0.012    0.045     0.005    1
          3     4     0.010    0.040     0.005    1
          3     6     0.004    0.040     0.005    1
          4     6     0.015    0.060     0.008    1
          4     9     0.018    0.070     0.009    1
          4    10     0.000    0.008     0.000    1
          5     7     0.005    0.043     0.003    1
          6     8     0.006    0.048     0.000    1
          7     8     0.006    0.035     0.004    1
          7    11     0.000    0.010     0.000    1
          8     9     0.005   0.048     0.000     1];

    lfybus                   % Forms the bus admittance matrix
    lfnewton     % Power flow solution by Newton-Raphson method
    busout       % Prints the power flow solution on the screen

%         Gen.  Ra   Xd'    H
gendata=[ 1     0   0.20   12
         10     0   0.15   10
         11     0   0.25   9];
trstab
```

Run the program for fault at bus 4 to obtain the swing plots for $t_c = 0.4$, and $t_c = 0.8$. Repeat until the critical clearing time $t_c = 0.72$ second is obtained.

CHAPTER 12 PROBLEMS

12.1. A 250-MW, 60-Hz turbine generator set has a speed regulation of 5 percent based on its own rating. The generator frequency decreases from 60 Hz to a steady state value of 59.7Hz. Determine the increase in the turbine power output.

The change in the angular frequency is

$$\Delta\omega = \frac{59.7 - 60}{60} = -0.005 \ \text{pu}$$

From (12.7), the change in the turbine power output is

$$\begin{aligned}\Delta P_g = -\frac{1}{R}\Delta\omega = (-\frac{1}{0.05})(-0.005) &= 0.10 \ \text{pu}\\ &= 25 \ \text{MW}\end{aligned}$$

12.2. Two generating units rated for 250 MW and 400 MW have governor speed regulation of 6.0 and 6.4 percent, respectively, from no-load to full-load, respectively. They are operating in parallel and share a load of 500 MW. Assuming free governor action, determine the load shared by each unit.

Expressing the governor speed regulation of each unit to a common MVA base of 1000, we get

$$\begin{aligned}R_1 &= \frac{1000}{250}(0.06) = 0.24\\ R_2 &= \frac{1000}{400}(0.064) = 0.16\end{aligned}$$

Since both units are operating at the same frequency, from (12.7) we have

$$R_1 P_{g1} = R_2 P_{g2}$$

or

$$P_{g2} = \frac{R_1}{R_2} P_{g1} = \frac{0.24}{0.16} P_{g1} = 1.5 P_{g1}$$

Also

$$P_{g1} + P_{g2} = P_D$$

Substituting for P_{g2} in the above equation, we have

$$P_{g1} + 1.5 P_{g1} = \frac{500}{1000}$$

which result in

$$P_{g1} = \frac{0.5}{2.5} = 0.2 \text{ pu} \qquad \text{or} \quad 200 \text{ MW}$$
$$P_{g2} = (1.5)(0.2) = 0.3 \text{ pu} \quad \text{or} \quad 300 \text{ MW}$$

12.3. A single area consists of two generating units, rated at 400 and 800 MVA, with speed regulation of 4 percent and 5 percent on their respective ratings. The units are operating in parallel, sharing 700 MW. Unit 1 supplies 200 MW and unit 2 supplies 500 MW at 1.0 per unit (60 Hz) frequency. The load is increased by 130 MW.
(a) Assume there is no frequency-dependent load, i.e., $D = 0$. Find the steady-state frequency deviation and the new generation on each unit.
(b) The load varies 0.804 percent for every 1 percent change in frequency, i.e., $D = 0.804$. Find the steady-state frequency deviation and the new generation on each unit.

Express the governor speed regulation of each unit to a common 1000MVA base, we have

$$R_1 = \frac{1000}{400}(0.04) = 0.1 \text{ pu}$$

$$R_2 = \frac{1000}{800}(0.05) = 0.0625 \text{ pu}$$

The per unit load change is

$$\Delta P_L = \frac{130}{1000} = 0.13 \text{ pu}$$

(a) From (12.15) with $D = 0$, the per unit steady-state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{-0.13}{10 + 16} = -0.005 \text{ pu}$$

Thus, the steady-state frequency deviation in Hz is

$$\Delta f = (-0.005)(60) = -0.30 \text{ Hz}$$

and the new frequency is

$$f = f_0 + \Delta f = 60 - 0.30 = 59.70 \text{ Hz}$$

The change in generation for each unit is

$$\begin{aligned}\Delta P_1 = -\frac{\Delta\omega}{R_1} = -\frac{-0.005}{0.10} &= 0.05 \text{ pu}\\ &= 50 \text{ MW}\end{aligned}$$

$$\begin{aligned}\Delta P_2 = -\frac{\Delta\omega}{R_2} = -\frac{-0.005}{0.0625} &= 0.08 \text{ pu}\\ &= 80 \text{ MW}\end{aligned}$$

Thus, unit 1 supplies 250 MW and unit 2 supplies 580 MW at the new operating frequency of 59.70 Hz.

(b) For $D = 0.804$, the per unit steady-state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2} + D} = \frac{-0.13}{10 + 16 + 0.804} = -0.00485 \text{ pu}$$

Thus, the steady-state frequency deviation in Hz is

$$\Delta f = (-0.00485)(60) = -0.291 \text{ Hz}$$

and the new frequency is

$$f = f_0 + \Delta f = 60 - 0.291 = 59.709 \text{ Hz}$$

The change in generation for each unit is

$$\begin{aligned}\Delta P_1 = -\frac{\Delta\omega}{R_1} = -\frac{-0.00485}{0.1} &= 0.0485 \text{ pu}\\ &= 48.5 \text{ MW}\end{aligned}$$

$$\Delta P_2 = -\frac{\Delta\omega}{R_2} = -\frac{-0.00485}{0.0625} = 0.0776 \text{ pu}$$
$$= 77.6 \text{ MW}$$

Thus, unit 1 supplies 248.5 MW and unit 2 supplies 577.6 MW at the new operating frequency of 59.775 Hz. The total change in generation is 126.1, which is 3.9 MW less than the 130 MW load change. This is because of the change in load due to frequency drop which is given by

$$\Delta\omega D = (-0.00485)(0.804) = -0.0039 \text{ pu}$$
$$= -3.9 \text{ MW}$$

12.4. An isolated power station has the LFC system as shown in Figure 97 with the following parameters

Turbine time constant $\tau_T = 0.5$ sec
Governor time constant $\tau_g = 0.25$ sec
Generator inertia constant $H = 8$ sec
Governor speed regulation = R per unit

The load varies by 1.6 percent for a 1 percent change in frequency, i.e., $D = 1.6$.
(a) Use the Routh-Hurwitz array (Appendix B.2.1) to find the range of R for control system stability.
(b) Use *MATLAB* **rlocus** function to obtain the root-locus plot.

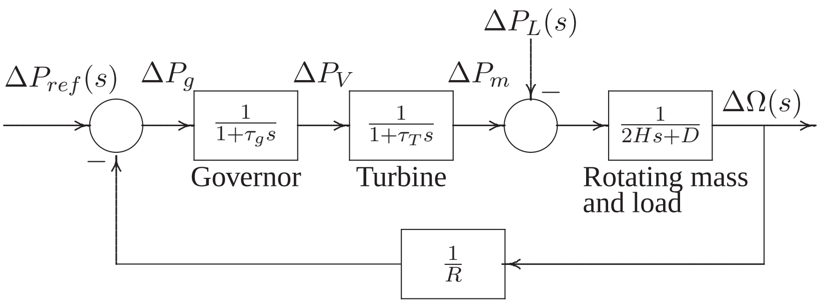


FIGURE 97
LFC system block diagram of Problem 12.4.

Redrawing the block diagram of Figure 97 with the load change $-\Delta P_L(s)$ as the input and the frequency deviation $\Delta\Omega(s)$ as the output and substituting the system parameters results in the block diagram shown in Figure 98. The open-loop transfer function is

$$KG(s)H(s) = \frac{K}{(16s + 1.6)(1 + 0.25s)(1 + 0.5s)}$$

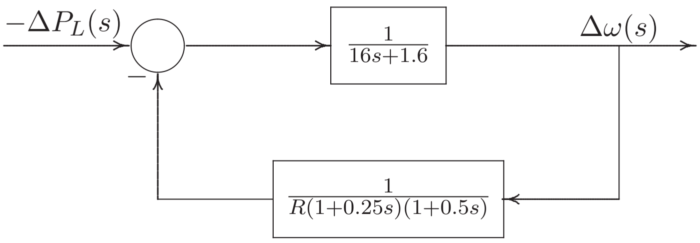


FIGURE 98
LFC block diagram for Problem 12.4.

$$= \frac{K}{2s^3 + 12.2s^2 + 17.2s + 1.6}$$

where $K = \frac{1}{R}$

(a) The characteristic equation is given by

$$1 + KG(s)H(s) = 1 + \frac{K}{2s^3 + 12.2s^2 + 17.2s + 1.6} = 0$$

which results in the characteristic polynomial equation

$$2s^3 + 12.2s^2 + 17.2s + 1.6 + K = 0$$

The Routh-Hurwitz array for this polynomial is then (see Appendix B.2.1)

$$\begin{array}{c|ll} s^3 & 2 & 17.2 \\ s^2 & 12.2 & 1.6+K \\ s^1 & \frac{206.64-2K}{12.2} & 0 \\ s^0 & 1.6+K & 0 \end{array}$$

From the s^1 row, we see that for control system stability, K must be less than 103.32. Also from the s^0 row, K must be greater than -1.6. Thus, with positive values of K, for control system stability

$$K < 103.32$$

Since $R = \frac{1}{K}$, for control system stability, the governor speed regulation must be

$$R > \frac{1}{103.32} \qquad \text{or} \qquad R > 0.009678$$

(b) Use the following commands to obtain the root-locus plot.

```
num=1;
den = [2  12.2  170.2 1.6];
figure(1), rlocus(num, den)
```

12.5. The governor speed regulation of Problem 12.4 is set to $R = 0.04$ per unit. The turbine rated output is 200 MW at nominal frequency of 60 Hz. A sudden load change of 50 MW ($\Delta P_L = 0.25$ per unit) occurs.
(a) Find the steady-state frequency deviation in Hz.
(b) Obtain the closed-loop transfer function and use *MATLAB* to obtain the frequency deviation step response.
(c) Construct the *SIMULINK* block diagram and obtain the frequency deviation response.

The closed-loop transfer function of the system shown in Figure 98 is

$$\frac{\Delta\Omega(s)}{-\Delta P_L(s)} = T(s) = \frac{(1+0.25s)(1+0.5s)}{(16s+1.6)(1+0.25s)(1+0.5s)+1/.04}$$
$$= \frac{0.0625s^2+0.375s+0.5}{s^3+6.1s^2+8.6s+13.3}$$

(a) The steady-state frequency deviation due to a step input is

$$\Delta\omega_{ss} = \lim_{s\to 0} s\Delta\Omega(s) = \frac{0.5}{13.3}(-0.25) = -0.0093985 \ \text{ pu}$$
$$= -0.5639 \ \text{ Hz}$$

(b) Use the following commands to obtain the step response.

```
PL = 0.25;
numc = [0.0625  0.375  0.5];
denc = [1   6.1  8.6  13.3];
t = 0:.02:10; c = -PL*step(num, den, t);
figure(2), plot(t, c), xlabel('t, sec'), ylabel('pu')
title('Frequency deviation step response'), grid
```

(c) Change the parameters of the *SIMULINK* model **sim12ex1.mdl** according to the values given in Problem 12.4 and run in the *SIMULINK* window to obtain the response.
12.6. The LFC system in Problem 12.5 is equipped with the secondary integral control loop for automatic generation control as shown in Figure 12.16 (Text).
(a) Use the *MATLAB* **step** function to obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.25$ per unit. Set the integral controller gain to $K_I = 9$.

(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).

(a) Substituting for the system parameters in (12.16), with speed regulation adjusted to $R = 0.04$ per unit, results in the following closed-loop transfer function

$$T(s) = \frac{0.0625s^3 + 0.375s^2 + 0.5s}{s^4 + 6.1s^3 + 8.6s^2 + 13.3s + 0.5K_I}$$

Use the following commands to obtain the step response.

```
PL = 0.25;
KI = 9;
num = [0.0625  0.375  0.5 0];
den = [1       6.1    8.6  13.3  0.5*KI];
t = 0:.02:12;
c = -PL*step(num, den, t);
plot(t, c), grid
xlabel('t, sec'), ylabel('pu')
title('Frequency deviation step response')
```

(b) Change the parameters of the *SIMULINK* model **sim12ex3.mdl** according to the values given in Problem 12.4 and run in the *SIMULINK* window to obtain the response.

12.7. The load changes of 200 MW and 150 MW occur simultaneously in areas 1 and 2 of the two-area system of Example 12.4. Modify the *SIMULINK* block diagram **sim12ex4.mdl**, and obtain the frequency and power deviation responses.

Apply the above load changes to the *SIMULINK* block diagram **sim12ex4.mdl**, and obtain the frequency deviation and the power responses (**sim12p7.mdl**).

12.8. Modify the *SIMULINK* model for the two-area system of Example 12.5 with the tie-line bias control (**sim12ex5.mdl**) to include the load changes specified in Problem 12.7. Obtain the frequency and power response deviation for each area.

Apply the load changes specified in Problem 12.7 to the *SIMULINK* block diagram (**sim12ex5.mdl**), and obtain the frequency and power deviation responses (**sim12p8.mdl**).

12.9. A generating unit has a simplified linearized AVR system as shown in Figure 99.
(a) Use the Routh-Hurwitz array (Appendix B.2.1) to find the range of K_A for

control system stability.
(b) Use *MATLAB* **rlocus** function to obtain the root-locus plot.
(c) The amplifier gain is set to $K_A = 40$. Find the system closed-loop transfer function, and use *MATLAB* to obtain the step response.
(d) Construct the *SIMULINK* block diagram and obtain the step response.

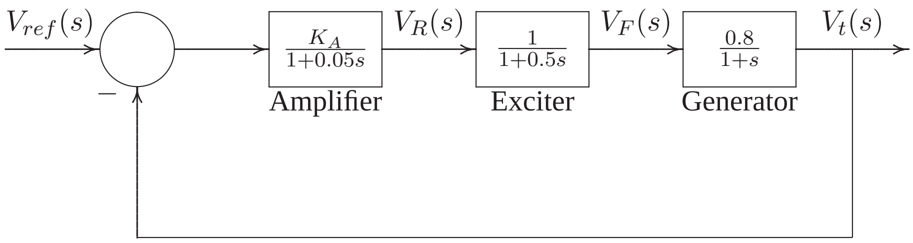


FIGURE 99
AVR system of Problem 12.9.

The open-loop transfer function of the AVR system shown in Figure 99 is

$$KG(s)H(s) = \frac{0.8K_A}{(1+0.05s)(1+0.5s)(1+s)}$$
$$= \frac{32K_A}{(s+20)(s+2)(s+1)}$$
$$= \frac{32K_A}{s^3+23s^2+62s+40}$$

(a) The characteristic equation is given by

$$1 + KG(s)H(s) = 1 + \frac{32K_A}{s^3+23s^2+62s+40} = 0$$

which results in the characteristic polynomial equation

$$s^3 + 23s^2 + 62s + 40 + 32K_A = 0$$

The Routh-Hurwitz array for this polynomial is then (see Appendix B.2.1)

s^3	1	62
s^2	23	$40 + 32K_A$
s^1	$\frac{1386-32K_A}{23}$	0
s^0	$40 + 32K_A$	

From the s^1 row we see that, for control system stability, K_A must be less than 43.3125, also from the s^0 row, K_A must be greater than -1.25. Thus, with positive values of K_A, for control system stability, the amplifier gain must be

$$K_A < 43.3125$$

(b) Use the following commands to obtain the root-locus plot.

```
num=32;
den=[1  23  62  40];
figure(1), rlocus(num, den);
```

(c) The closed-loop transfer function of the system shown in Figure 99 for $K_A = 40$ is

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{1280}{s^3 + 23s^2 + 62s + 1320}$$

(c) Use the following commands to obtain the step response.

```
numc= 1280;
denc=[1  23  62  1320];
step(numc, denc);
```

(d) Construct a *SIMULINK* block diagram model for Figure 99, and simulate to obtain the response (**sim12p9.mdl**).

12.10. A rate feedback stabilizer is added to the AVR system of Problem 12.9 as shown in Figure 100. The stabilizer time constant is $\tau_F = 0.04$ second, and the derivative gain is adjusted to $K_F = 0.1$.
(a) Find the system closed-loop transfer function, and use *MATLAB* to obtain the step response.
(b) Construct the *SIMULINK* model, and obtain the step response.

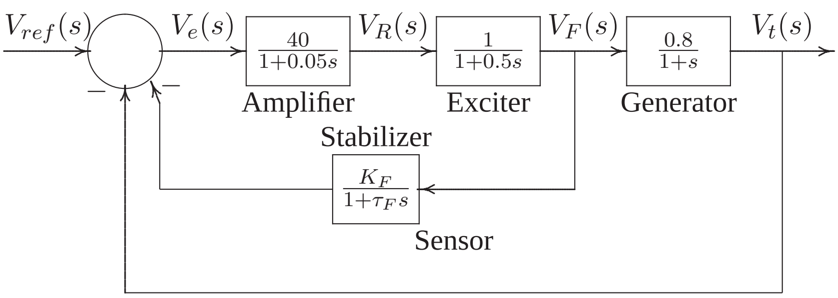


FIGURE 100
AVR system with rate feedback for Problem 12.10.

(a) The system closed-loop transfer function is

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{\frac{32}{(1+.05s)(1+0.5s)(1+s)}}{1 + \frac{4}{(1+.05s)(1+0.5s)(1+0.04s)} + \frac{32}{(1+.05s)(1+0.5s)(1+s)}}$$

$$= \frac{1280(s+25)}{s^4 + 48s^3 + 637s^2 + 6870s + 37000}$$

(a) Use the following commands to obtain the step response.

```
numc=1280*[1 25];
denc=[1  48  637  6870 37000];
t=0:.02:3;
step(numc, denc, t), grid
xlabel('t, sec.'), title('Terminal voltage step response')
```

(b) Construct a *SIMULINK* block diagram model for Figure 100, and simulate to obtain the response (**sim12p10.mdl**).

12.11. A PID controller is added in the forward path of the AVR system of Problem 12.9 as shown in Figure 101. Construct the *SIMULINK* model. Set the proportional gain K_P to 2.0, and adjust K_I and K_D until a step response with a minimum overshoot and a very small settling time is obtained (suggested values $K_I = 0.15$, and $K_D = 0.17$).

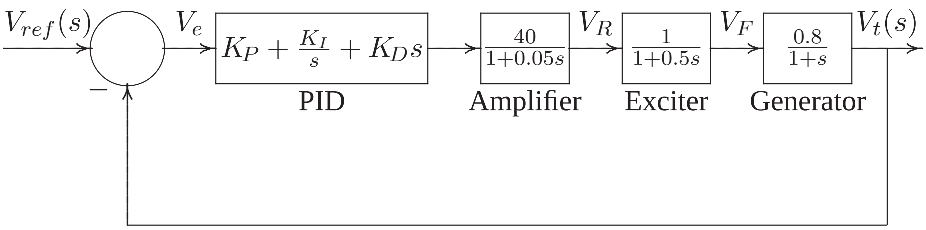


FIGURE 101
AVR system with PID controller for Problem 12.11.

Construct a *SIMULINK* block diagram model for Figure 101, set $K_P = 1$, $K_I = 0.15$, and $KD = 0.17$, simulate to obtain the response (**sim12p11.mdl**).

12.12. Figure 102 shows an inverted pendulum of length L and mass m mounted on a cart of mass M, by means of a force u applied to the cart. This is a model of the attitude control of a space booster on takeoff. The differential equations describing the motion of the system is obtained by summing the forces on the pendulum, which result in the following nonlinear equations.

$$(M+m)\ddot{x} + mL\cos\theta\,\ddot{\theta} = mL\sin\theta\,\dot{\theta}^2 + u$$
$$mL\cos\theta\,\ddot{x} + mL^2\ddot{\theta} = mgL\sin\theta$$

(a) Linearize the above equations in the neighborhood of the zero initial states.

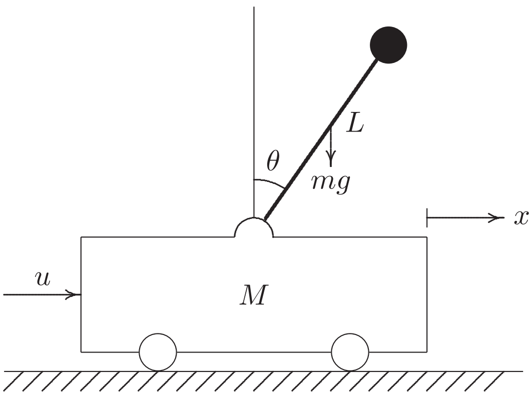


FIGURE 102
Inverted pendulum on a cart.

Hint: Substitute θ for $\sin\theta$, 1 for $\cos\theta$ and 0 for $\dot{\theta}^2$. With the state variables defined as $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = x$, and $x_4 = \dot{x}$, show that the linearized state equation is

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M+m}{ML}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M}g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{ML} \\ 0 \\ \frac{1}{M} \end{bmatrix} u$$

Assume $M = 4$ kg, $m = 0.2$ kg, $L = 0.5$ m, and $g = 9.81$ m/s^2.
(b) Use the *MATLAB* function **eig(A)** to find the roots of the system characteristic equation.
(c) Define C as the identity matrix of rank 4, i.e., $C = \text{eye}(4)$ and $D = \text{zeros}(4, 1)$. Use the *MATLAB* function $[\mathbf{y}, \mathbf{x}] = \text{initial}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{X_0}, \mathbf{t})$ to simulate the system for 20 seconds in response to an initial condition offset of $\theta(0) = 0.1$ rad, and $x(0) = 0.1$ m (i.e., $x_0 = [0.1 \quad 0 \quad 0.1 \quad 0]$). Obtain a plot of θ and x, and comment on the stability of the system.
(d) You may have found that the inverted pendulum is unstable, that is, it will fall over unless a suitable control force via state feedback is used. The purpose is to design a control system such that for a small initial disturbance the pendulum can be brought back to the vertical position ($\theta = 0$), and the cart can be brought back to the reference position ($x = 0$). A simple method is to use a state feedback gain to place the compensated closed-loop poles in the left-half of the s-plane. Use the custom made function **[K, Af] = placepol(A, B, C, P)** and design a state feedback controller to place the compensated closed-loop poles at $-2 \pm j0.5$, -4, and -5. Simulate the system for 20 seconds in response to the same initial condition offset. Obtain a plot of θ, x, and $u = -Kx'$.

The above nonlinear equations can be linearized provided the angle θ is sufficiently small. Substituting θ for $\sin\theta$, 1 for $\cos\theta$ and 0 for $\dot{\theta}^2$, we obtain the following equations

$$(M+m)\ddot{x} + mL\,\ddot{\theta} = u$$
$$mL\ddot{x} + mL^2\ddot{\theta} = mgL\theta$$

Defining the following state variables as

$$x_1 = \theta \qquad x_2 = \dot{\theta} \qquad x_3 = x \qquad x_4 = \dot{x}$$

then, the state variable derivatives are

$$\dot{x_1} = x_2 \qquad \dot{x_2} = \ddot{\theta} \qquad \dot{x_3} = x_4 \qquad \dot{x_4} = \ddot{x}$$

Substituting for $\ddot{x}$, and $\ddot{\theta}$ in the linearized equations, result in

$$(M+m)\dot{x_4} + mL\,\dot{x_2} = u$$
$$mL\dot{x_4} + mL^2\dot{x_2} = mgLx_1$$

Solving for $\dot{x_2}$ and $\dot{x_4}$, we have

$$\dot{x_2} = \frac{M+m}{ML}gx_1 - \frac{1}{ML}u$$
$$\dot{x_4} = -\frac{m}{M}gx_1 + \frac{1}{M}u$$

and the linearized state space equation becomes

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M+m}{ML}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M}g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{ML} \\ 0 \\ \frac{1}{M} \end{bmatrix} u$$

We use the following commands

```
M = 4; m = 0.2; g =9.81; L =.5;
A = [      0          1   0    0
     (M+m)/(M*L)*g  0   0    0
           0          0   0    1
       -m/M*g         0   0    0];
B = [0; -1/(L*M);   0;  1/M];
disp('(b) Roots of the characteristic equation are')
r = eig(A)
disp('(c)')
```

```
C = eye(4);
D = zeros(4, 1);
t=0:0.01:2.;
X0=[0.1 0  0.1  0];
[y, x] = initial(A, B, C, D, X0, t);
figure(1),plot(t, x(:,1), t, x(:, 3)), grid
xlabel('t, sec')
disp('(d)')
P =[-2+j*.5,  -2-j*.5,  -5,  -4];
[K, Af] = placepol(A, B, C, P)
t=0:0.1:10;
[y, x] = initial(Af, B, C, D, X0, t);
figure(2),plot(t, x(:,1), t, x(:, 3)), grid
xlabel('t, sec')
u = -K*x';
figure(3),plot(t, u), grid
xlabel('t, sec.')
```

The result is

```
(b) Roots of the characteristic equation are
    r =

        0
        0
        4.5388
       -4.5388
(c), (d)
    Feedback gain vector K
      -170.367   -38.054   -17.3293   -24.1081
    K =
        -170.3666  -38.0540  -17.3293  -24.1081
    Af =

         0          1.0000    0          0
       -64.5823  -19.0270   -8.6646  -12.0540
         0          0         0          1.0000
        42.1012    9.5135    4.3323    6.0270
```

Run the program to obtain the response. In part (c) the response is unstable, and in part(d), with state feedback we have a stable response.

12.13. Construct the *SIMULINK* block diagram for the linearized model of the inverted pendulum described in Problem 12.12 (a) with the state feedback controller. Assume the state feedback gains are $K_1 = -170.367$, $K_2 = -38.054$, $K_3 =$

-17.3293, and $K_4 = -24.1081$. Obtain the response for θ, x, and u for the initial condition offset of $\theta(0) = 0.1$ rad and $x(0) = 0.1$m (i.e., $x_0 = [0.1 \quad 0 \quad 0.1 \quad 0]$).

Construct the *SIMULINK* block diagram shown in Figure 103 (**sim12p13.mdl**).

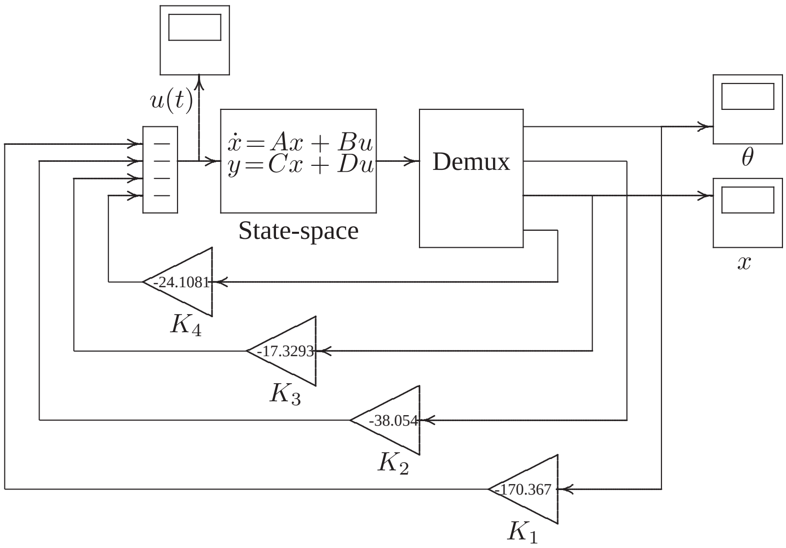


FIGURE 103
Simulation block diagram for Problem 12.13.

Open the State-Space block, and set the parameters to

$$A = [0 \quad 1 \quad 0 \quad 0;\ 20.601 \quad 0 \quad 0 \quad 0;\ 0 \quad 0 \quad 0 \quad 1;\ -0.4905 \quad 0 \quad 0 \quad 0]$$
$$B = [0 \quad -0.5 \quad 0 \quad 0.25]$$
$$C = eye(4)$$
$$D = zeros(4,1)$$

Simulate to obtain the responses $u(t), \theta(t)$, and $x(t)$.

12.14. A classical problem in control systems is to find the optimal control law which will transfer the system from its initial state to the final state, such that a given performance index is minimized.
(a) Design an LQR state feedback for the linearized model of the inverted pendulum described in Problem 12.12, and find the optimal feedback gain vector to minimize the performance index

$$\mathbf{J} = \int_0^\infty \left(10{x_1}^2 + 10{x_2}^2 + 5{x_3}^2 + +5{x_4}^2 + 0.2u^2\right) dt$$

The admissible states and control values are unconstrained.
(b) Define C as the identity matrix of rank 4, i.e., $C = \text{eye}(4)$ and $D = \text{zeros}(4, 1)$. Use the *MATLAB* function **[K, P] = lqr2 (A, B, Q, R)** to design a state feedback controller in response to an initial condition offset of $\theta(0) = 0.1$ rad and $x(0) = 0.1$m (i.e., $x_0 = [0.1 \quad 0 \quad 0.1 \quad 0]$). Use the *MATLAB* function $[\mathbf{y}, \mathbf{x}] = \text{initial}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{X_0}, \mathbf{t})$ to simulate the system for 20 seconds. Obtain a plot of θ, x, and the control law $u = -kx'$.

The following statements

```
M = 4; m = 0.2; g =9.81; L =.5;
A = [     0          1   0    0
     (M+m)/(M*L)*g  0   0    0
          0         0   0    1
       -m/M*g       0   0    0];
B = [0; -1/(L*M);   0;  1/M]; C = eye(4); D = zeros(4, 1);
Q = [10   0    0   0
      0  10   0   0
      0   0   5   0
      0   0    0  5];
R = .2;
[K, P] = lqr2(A, B, Q, R)
Af = A - B*K
t=0:0.1:10; X0=[0.1 0  0.1  0];
[y, x] = initial(Af, B, C, D, X0, t);
figure(1),plot(t, x(:,1), t, x(:, 3)), grid
xlabel('t, sec'), ylabel('pu')
u = -K*x';
figure(2),plot(t, u), grid, xlabel('t, sec'), ylabel('pu')
```

result in

```
    K =
       -125.0988  -28.6369   -5.0000  -10.5129
    P =
        365.3748   77.0073   28.6369   53.9355
         77.0073   17.3230    6.2761   11.7364
         28.6369    6.2761   10.5129    8.5522
         53.9355   11.7364    8.5522   15.0625
Af =
         0          1.0000    0          0
       -41.9484  -14.3185   -2.5000   -5.2565
         0          0         0          1.0000
        30.7842    7.1592    1.2500    2.6282
```

Run the program to obtain the plots.

12.15. Construct the *SIMULINK* block diagram for the linearized model of the inverted pendulum described in Problem 12.12(a) using the *SIMULINK* LQR model. Obtain the response for θ, x, and u for the initial condition offset described in Problem 12.14.

Construct the *SIMULINK* block diagram shown in Figure 104.

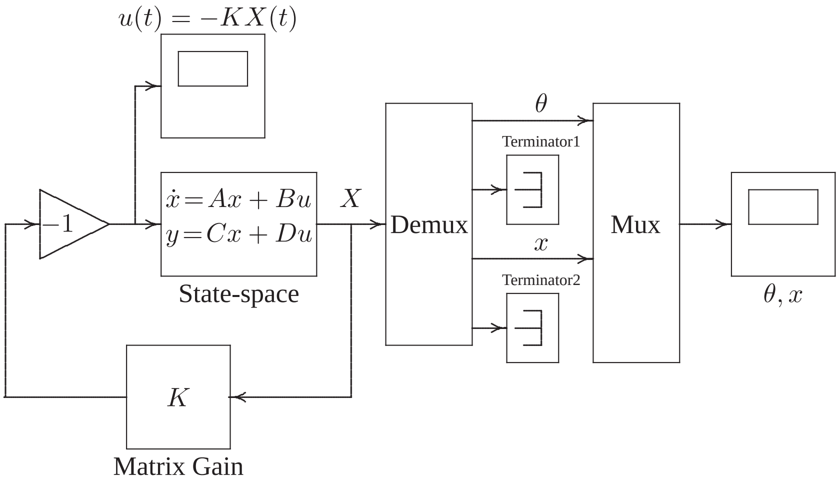


FIGURE 104
Simulation block diagram for Problem 12.15.

Open the State-Space block, and set the parameters to

$$A = [0 \quad 1 \quad 0 \quad 0;\ 20.601 \quad 0 \quad 0 \quad 0;\ 0 \quad 0 \quad 0 \quad 1;\ -0.4905 \quad 0 \quad 0 \quad 0]$$
$$B = [0 \quad -0.5 \quad 0 \quad 0.25]$$
$$C = eye(4)$$
$$D = zeros(4, 1)$$

Also, open the Matrix Gain dialog box and set K to

$$K = [-170.367 \quad -38.054 \quad -17.3293 \quad -24.1081]$$

Simulate to obtain the responses $u(t), \theta(t)$, and $x(t)$.

12.16. Obtain the state variable representation of the LFC system of Problem 12.4 with one input ΔP_L, and perform the following analysis.
(a) Use the *MATLAB* **step** function to obtain the frequency deviation step response

for a sudden load change of $\Delta P_L = 0.2$ per unit.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).
(c) Use **placepol(A, B, C, p)** function to place the compensated closed-loop pole at $-4 \pm j6$ and -4. Obtain the frequency deviation step response of the compensated system.
(d) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (c).

From (12.54), the state equation of the LFC system of Problem 12.4 is

$$\begin{bmatrix} \Delta \dot{P}_V \\ \Delta \dot{P}_m \\ \Delta \dot{\omega} \end{bmatrix} = \begin{bmatrix} -4 & 0 & -100 \\ 2 & -2 & 0 \\ 0 & 0.0625 & -0.1 \end{bmatrix} \begin{bmatrix} \Delta P_V \\ \Delta P_m \\ \Delta \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.0625 \end{bmatrix} \Delta P_L$$

and the output equation is

$$y(t) = [0 \quad 0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

For part (a) and (c), we use the following statements

```
PL = 0.2;
A = [-4  0  -100; 2  -2  0; 0  0.0625  -0.1];
B = [0;  0;  -0.0625]; BPL = B*PL;
C = [0  0  1];  D = 0;
disp('(a)')
t=0:0.02:10;
[y, x] = step(A, BPL, C, D, 1, t);
figure(1), plot(t, y), grid
xlabel('t, sec'), ylabel('pu')
r =eig(A)
disp('(c) Pole-placement design')
P=[-4+j*6   -4-j*6   -4];
[K, Af] = placepol(A, B, C, P);
t=0:0.02:4;
[y, x] = step(Af, BPL, C, D, 1, t);
figure(2), plot(t, y), grid
xlabel('t, sec'), ylabel('pu')
```

The result is

```
(a)
```

```
    r =
       -4.8987
       -0.6007 + 1.5343i
       -0.6007 - 1.5343i

c) Pole-placement design
    Feedback gain vector K
        6.4   5.4   -94.4

    Uncompensated Plant
    Transfer function:
    -0.0625 s^2 - 0.375 s - 0.5
    ----------------------------
    s^3 + 6.1 s^2 + 8.6 s + 13.3

    Compensated system closed-loop
    Transfer function:
    -0.0625 s^2 - 0.375 s - 0.5
    ---------------------------
    s^3 + 12 s^2 + 84 s + 208

    Compensated system matrix A - B*K
       -4.0000         0 -100.0000
        2.0000   -2.0000         0
        0.4000    0.4000   -6.0000
```

Run the program to obtain the frequency deviation step responses.

The simulation block diagram for part (b) and (d) is shown in Figure 105 (**sim12p16.mdl**).

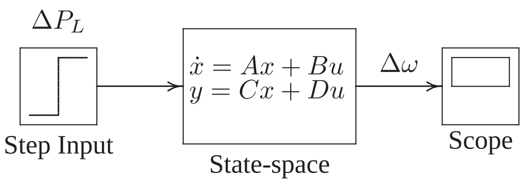


FIGURE 105
Simulation block diagram for Problem 12.16.

(b) Open the State-Space block, and set the parameters to

$$A = [-4 \quad 0 \quad -100;\ 2 \quad -2 \quad 0;\ 0 \quad 0.0625 \quad -0.1]$$
$$B = 0.2 * [0 \quad 0 \quad 0.06255]$$

$$C = [0 \quad 0 \quad 1]$$
$$D = 0$$

Simulate to obtain the frequency deviation response.

(d) Change the A matrix to

$$A = [-4 \quad 0 \quad -100;\ 2 \quad -2 \quad 0;\ 0 \quad 0.0625 \quad -0.1]$$

Simulate to obtain the compensated frequency deviation response.

12.17. Design a LQR state feedback for the system described in Problem 12.16.
(a) Find the optimal feedback gain vector to minimize the performance index

$$\mathbf{J} = \int_0^\infty \left(40{x_1}^2 + 20{x_2}^2 + 10{x_3}^2 + 0.2u^2\right) dt$$

The admissible states and control values are unconstrained. Obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.2$ per unit.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation step response for the condition in part (a).

For this system we have

$$\mathbf{A} = \begin{bmatrix} -4 & 0 & -100 \\ 2 & -2 & 0 \\ 0 & 0.0625 & -0.1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -0.0625 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

and $R = 0.20$.

(a) We use the following commands

```
PL=0.2;
A = [-4  0  -100; 2  -2  0; 0  0.0625  -0.1];
B = [0;  0;  -0.0625]; BPL=PL*B;
C = [0  0  1]; D = 0;
Q = [40 0  0; 0  20  0; 0  0  10];
R = .2;
[K, P] = lqr2(A, B, Q, R)
Af = A - B*K
t=0:0.02:1;
[y, x] = step(Af, BPL, C, D, 1, t);
plot(t, y), grid
xlabel('t, sec'), ylabel('pu')
```

The result is

```
K =
     8.3477    1.4637 -162.0007
P =
     3.7062    0.8966  -26.7126
     0.8966    4.7465   -4.6839
   -26.7126   -4.6839  518.4023
Af =
    -4.0000         0 -100.0000
     2.0000   -2.0000         0
     0.5217    0.1540  -10.2250
```

Run the program to obtain the frequency deviation step response.

(b) A *SIMULINK* model is shown in Figure 106. Open the state-space description dialog box, and enter the $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ constants in the appropriate box. Also, open the LQR description dialog box, and set the weighting matrix $\mathbf{Q}$ and weighting coefficient $\mathbf{R}$ to the given values. Run in the *SIMULINK* window and obtain

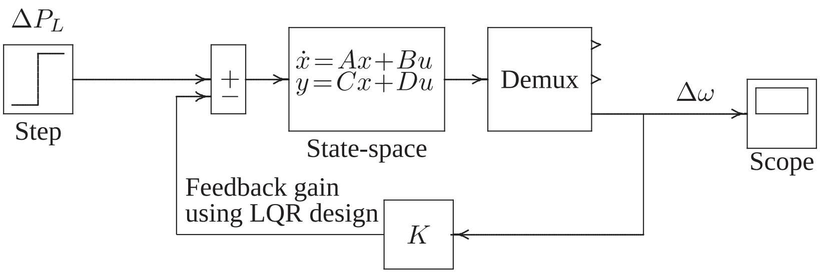


FIGURE 106
Simulation block diagram for Problem 12.17.

the simulation response.